A Semi-analytical Description for the Formation and Gravitational Evolution of Protoplanetary Disks

Sanemichi Z. Takahashi (Kyoto University/Nagoya University),
Shu-ichiro Inutsuka (Nagoya University), Masahiro N. Machida(Kyushu University)

We investigate the formation process of self-gravitating protoplanetary disks.The angular momentum in the disks is redistributed by the action of gravitational torque in the massive disk in its early formation stage. We develop a simplified one-dimensional accretion disk models that take into account the infalling gas from the envelope onto the disk and the transfer of angular momentum within the disk in terms of effective viscosity. We find that the disk evolution does not depend on the detail of modeling for effective viscosity. The structures of resultant disks agree with the results of three dimensional simulations.

1. Introduction

- The process of the star and planets formation

Star Formation

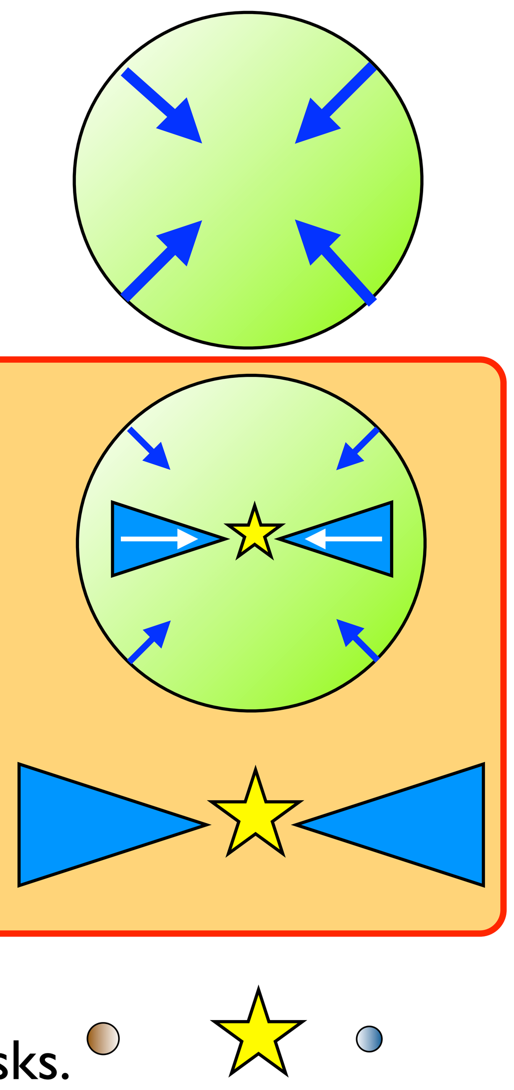
Star formation start with the gravitational collapse of the cloud core.

A protostar and a circumstellar disk are formed .

All of gas accrete onto the disk.
→Protoplanetary disk

Planet Formation

Planets are formed in the protoplanetary disks.



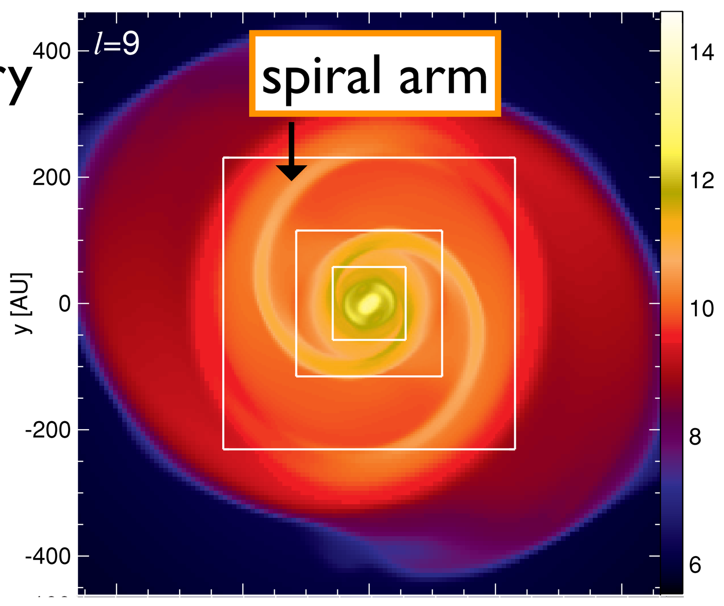
Protoplanetary disks provide an initial condition of the planet formation .

→ **We investgate the formation process of protoplanetary disks.**

2. Gravitational evolution of the disks

In the early formation stage, the protoplanetary disk is more massive than the protostar.

→ gravitationally unstable
→ Spiral arm is formed.
→ nonaxisymmetric gravity
→ gravitational torque
→ angular momentum is redistributed



Machida et al. (2010)

The process depends on the detail of the spiral arm.
How to model the angular momentum transfer?

➡ We test the effective viscosity model.

3. Calculations

Mass conservation

$$\frac{\partial}{\partial t} 2\pi r \Sigma = -\frac{\partial F}{\partial r} + g_{acc}$$

Σ:surfase density
F: mass flax
Gas accretion from the cloud core

Angular momentum conservation

$$\frac{\partial}{\partial t}(2\pi r \Sigma j) = -\frac{\partial}{\partial r}\left(F j + 2\pi \Sigma r^3 \nu \frac{\partial \Omega}{\partial r}\right) + g_{acc} j_{acc}$$

Ω : angular velocity
j : specific angular momentum
c_s : sound speed

α disk model

$$\nu = \alpha c_s^2 / \Omega$$

$$\alpha = a \exp(-bQ^4) + 0.01$$ (cf. Zhu et. al. 2010)

Force balance between the centrifugal force and gravity

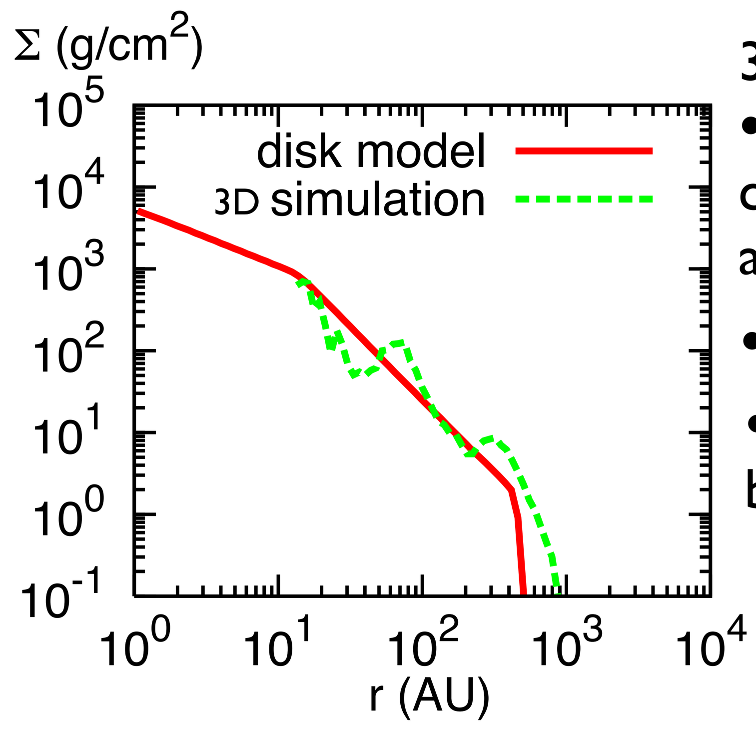
$$\frac{j^2}{r^3} = \frac{G M_r}{r^2}$$

M_r : total mass enclosed within radius r

- Initial conditions : protosar mass 10^{-2} Msun, zero disk mass
- Cloud core: mass 2.5Msun, angular velocity $\Omega_0 = 4.8\times10^{-14}$ s^{-1}

4. Results

Comparing with 3D simulation

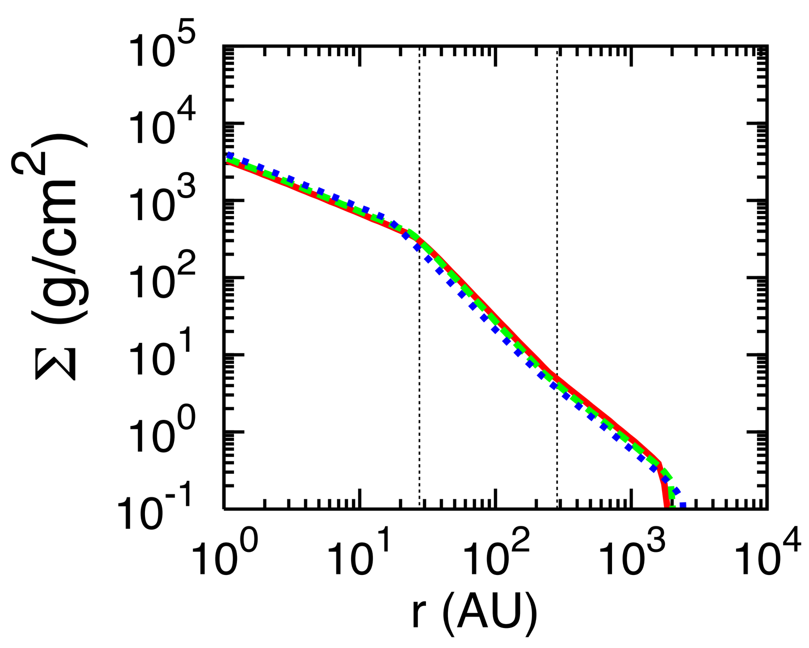


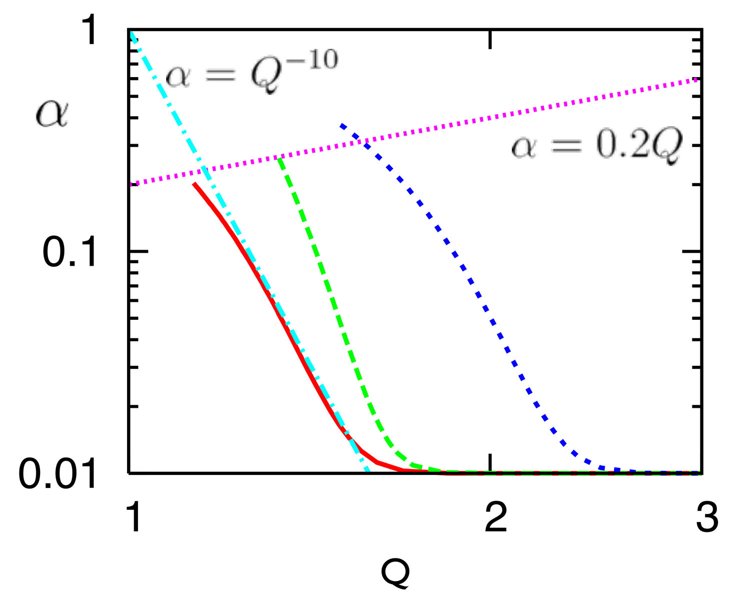
3D simulation :

- calculate from gravitational collapse of cloud core to formation and evolution of protostar and protopranetary disk.
- 10^5 years after the protostar is formed
- Surface density distribution is waggly because of spiral arms.

The disk has the structure that is in **agreement** with the result of 3D simulation.

Dependence on effective viscosity





3.5x10^5 year after the protostar is formed.
All of the gas accrete onto the disk.

model A (a,b)=(1, 1)
model B (a,b)=(6, 1)
model C (a,b)=(1, 0.2)

No large difference in the surface densities of 3 models.

mass flux
(~ mass accretion rate)

$$F = \frac{1}{\frac{\partial j}{\partial r}} \frac{\partial}{\partial r}\left((2\pi r \Sigma)\nu r^2 \frac{\partial \Omega}{\partial r}\right)$$

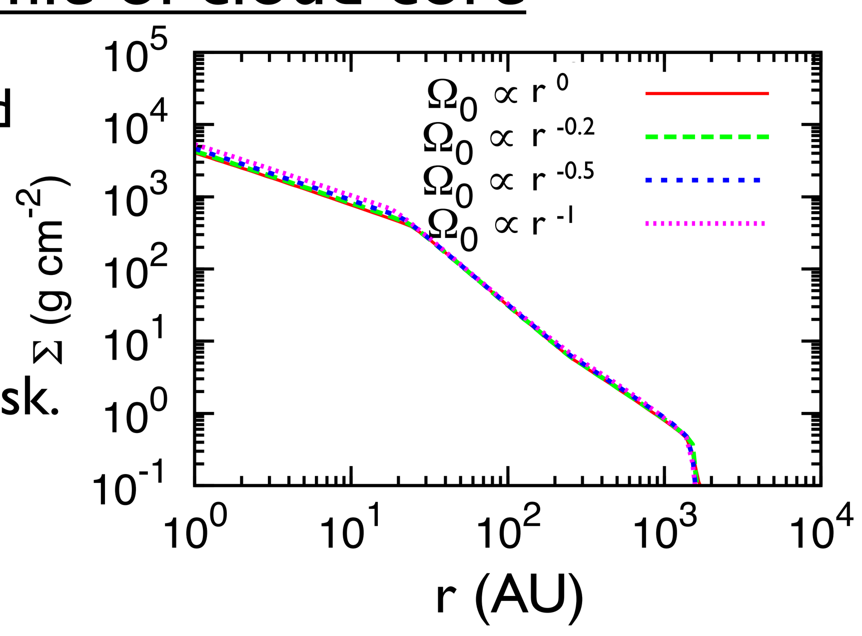
➡ $$\alpha \simeq 0.2Q \left(\frac{F}{2\times10^{-6} M_\odot/\mathrm{yr}}\right)\left(\frac{v_\phi^2/c_s^2}{100}\right)^{\frac{3}{2}}\left(\frac{r}{300\mathrm{AU}}\right)^{\frac{3}{2}}\left(\frac{M_r}{1M_\odot}\right)^{-\frac{3}{2}}$$

The model in which α changes drastically when Q changes slightly
→ The structure of the disk is **self-regulated**.

Dependence on rotation profile of cloud core

Angular momentum is well redistributed by the gravitational torque.

➡ Rotation profile of the cloud core produces only a small effect on the disk.



5. Summary

- Formation and evolution of protoplanetary disks in the early stage is investigated using the effective viscosity model.
- The disk evolution is self-regulated and does not depend on the detail of modeling for effective viscosity.
- The structures of resultant disks agree with the results of three dimensional simulations.
- Each run takes only about 10 seconds in laptop computers!