Nonlinear Ohm's Law: Electric-Field Heating of Plasmas in PPDs

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Magnetic resistivity plays crucial roles in MHD of protoplanetary disks. It is commonly assumed that electric fields (in the neutral-gas rest frame) are so weak that they have no effect on the kinetics of ionized gases. However, an estimate shows that the E-field induced by MRI turbulence can become strong enough to heat up free electrons (Inutsuka & Sano 2005; see also below).

We construct a simple charge-reaction model that takes into account the E-field heating of weakly ionized plasmas as well as the impact ionization of the gas by hot electrons. Our model calculations show that the heating promotes the adsorption of electrons onto dust grains, resulting in an enhancement of the resistivity up to factor 10^3. This "anomalous" resistivity may perhaps determine the saturation level of MRI-driven turbulence. We also find that a very high E-field causes an abrupt increase in the electric current ("electron avalanche") as predicted by Inutsuka & Sano. However, this solution is found to be unstable in some cases.

Plasma Heating in MRI Turbulence

In weakly ionized gases, the random velocity of plasma particles (ions and electrons) is determined by the balance between collisional energy/momentum transfer with neutrals and acceleration by an applied electric field E'. If E' is small, collisional cooling approximately balances with collisional heating, and hence the E-field has no effect on the energy distribution functions of the charged particles. This is the situation where ***linear*** Ohm's law holds.

However, if E' is so high that E-field heating dominates over collisional heating, then the random velocities of the charged particles (mainly electrons) increase with E'. This leads to ***nonlinear*** Ohm's law as the charge reaction rates now depend on E'. The criterion for this to occur is given by (see Landau & Lifshitz 1994, *Physical Kinetics*)

$$E \gtrsim E_{\rm crit} \equiv \sqrt{\frac{6m_e}{m_n}}\frac{k_{\rm B}T}{e\ell_e} \approx 10^{-9}\left(\frac{T}{100\ {\rm K}}\right)\left(\frac{n_n}{10^{12}\ {\rm cm^{-3}}}\right)\ {\rm esu\ cm^{-2}}$$

where T and n_n are the temperature and number density of the neutrals, respectively, and l_e is the mean free path of the electrons. The factor m_e/m_n accounts for the low energy-transfer efficiency of electron–neutral collisions.

We compare $E_{\rm crit}$ with the typical E-field strength in MRI turbulence. MHD simulations show that the RMS current density in MRI-driven turbulence scales as (Muranushi, Okuzumi, & Inutsuka 2012)

$$\langle J^2\rangle^{1/2}_{\rm MRI} \approx f_{\rm sat}\sqrt{\frac{\rho_g}{8\pi}}c\Omega$$

where ρ is the gas density, c is the speed of light, and Ω is the orbital frequency, and $f_{\rm sat} \sim 10$ is a numerical factor. This can be recast into a relation for the RMS E-field strength in the neutral-rest frame,

$$\langle E'^2\rangle^{1/2}_{\rm MRI} \approx \sqrt{\frac{8\pi n_n}{m_n}}\frac{f_{\rm sat}k_{\rm B}T}{c\beta_z\Lambda_z} \approx 10^{-7}\Lambda_z^{-1}\left(\frac{10^2}{\beta_z}\right)\left(\frac{f_{\rm sat}}{10}\right)\left(\frac{T}{100\ {\rm K}}\right)\left(\frac{n_n}{10^{-12}\ {\rm cm^{-3}}}\right)^{-1/2}\ {\rm esu\ cm^{-2}}$$

where $\Lambda_z = v_{Az}^2/\eta\Omega$ is the Ohmic Elsasser number and $\beta_z = 2c_s^2/v_{Az}^2$. The RMS field strength exceeds $E_{\rm crit}$ when

$$\Lambda_z \lesssim 10^2\left(\frac{f_{\rm sat}}{10}\right)\left(\frac{10^2}{\beta_z}\right)\left(\frac{n_n}{10^{12}\ {\rm cm^{-3}}}\right)^{-1/2}$$

Since MRI operates at $\Lambda_z > 0.1$–1, we fid that ***E-field heating should occur near dead-zone edges*** where 0.1–$10 < \Lambda_z < 10^2$. Note that the field heating is particularly relevant when β_z is low, i.e., *when MRI-turbulence is strong.*

A Charge-Reaction Network including E-Field Heating

We calculate the ionization balance of a gas–dust mixture taking into account E-field heating (Fig. 1). For weakly ionized gases, the velocity distributions of charged particles can be analytically derived (Landau & Lifshitz 1994; see also Fig. 2). These allow to calculate the rate coefficients for charge reactions, and in turn the ionization balance, as a function of E'. Combining this with the plasma drift velocities, we obtain the current density J as a function of E' (Ohm's law). Our charge reactions include impact ionization by hot electrons (red arrows in Fig. 2).

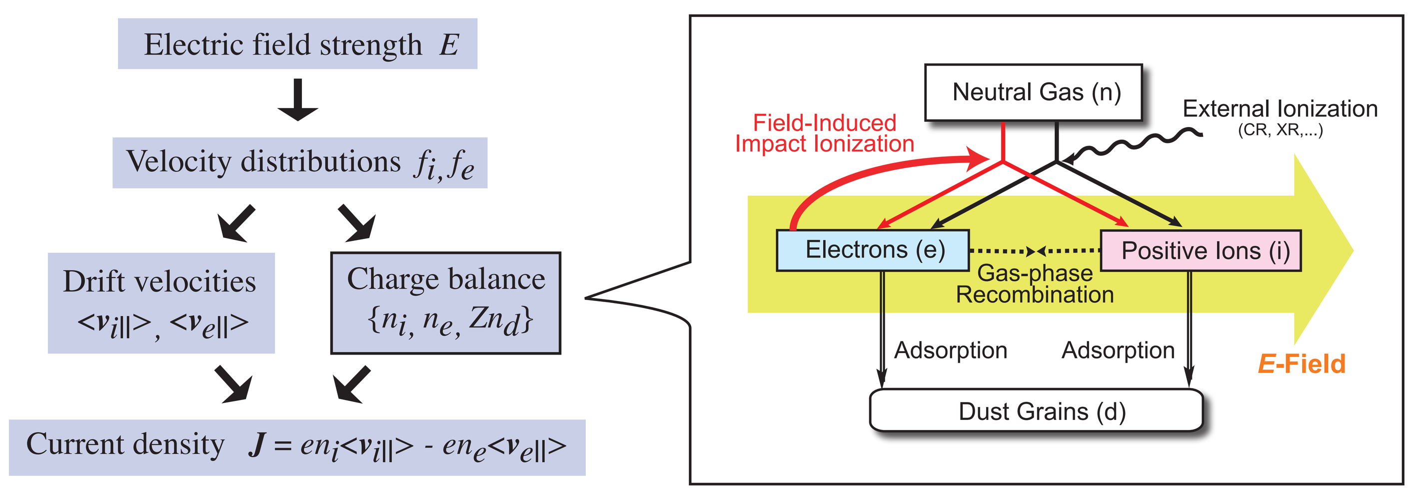

Fig 1. Flow chart of the model that provides the current density as a function of the E-field strength.

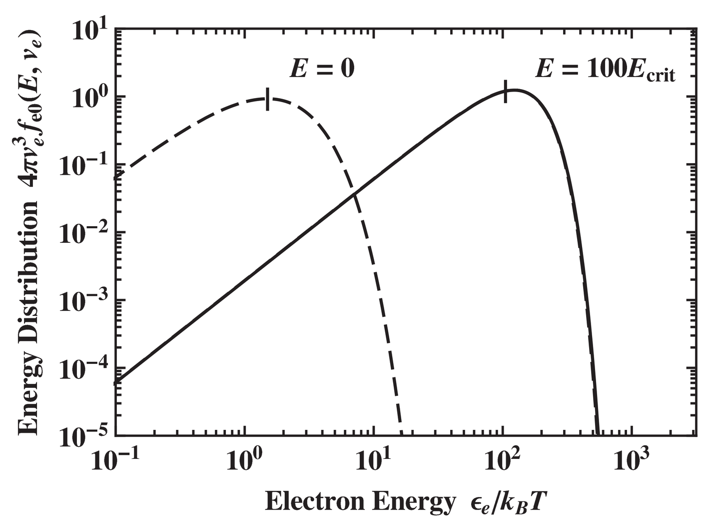

Fig 2. Energy distribution function of electrons for $E' = 0$ (dashed curve) and $E' = 100E_{\rm crit}$ (solid curve).

Model Calculations

Gas: $n_n = 10^{12}\ {\rm cm^{-3}}$; $T = 100$ K; $\zeta = 10^{-17}\ {\rm s^{-1}}$

Dust: ${\rm d/g} = \left\{\begin{array}{l}10^{-4}\ ({\bf B})\\ 10^{-2}\ ({\bf C})\end{array}\right\}$; $a = 1\ \mu{\rm m}$; $\rho_{\rm mat} = 2\ {\rm g\ cm^{-3}}$

The derived Ohm's laws are nonlinear at $E' > E_{\rm crit}$ (in the right of the gray vertical lines).

The left panels show that the electron abundance x_e decreases as E' is increased from $E_{\rm crit}$. This occurs because heated electrons more frequently collide with and adsorb onto dust grains. This leads to a decrease in the current density J, as seen in the middle and right panels.

The sharp increase in the plasma abundances and current density at higher E' is due to the impact ionization by high-energy electrons (see also Inutsuka & Sano 2005). This "avalanche" current is, however, found to be unstable against charge perturbation when d/g is high (see Model C).

The right panels show the resulting Ohmic resistivity η as a function of E'. We see that η increases with E' by up to factor 10^3. If this occurs in MRI turbulence, this will act as an anomalous resistivity, which will increase the size of dead zones and might even determine the saturated level of MRI turbulence. In future work, we will address these questions and will also extend our analysis to non-Ohmic resistivities.

Plasma Abundances | **Current Density** | **Resistivity** (Ohmic)