



# Satellite formation from ancient massive rings



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**Context :** A *tidal disk* (disk of solids around a planet, in which tides prevent accretion) spreads beyond the Roche radius  $r_R$ . What happens ?

**Result :** First, 1 satellite accretes all the incoming material, until a critical mass. Then, a series of satellites form, migrate, and merge.

**Conclusion :** This *analytical* model explains the structure of Saturn's system, but also applies to Uranus, Neptune... and the Earth !

## Prerequisites and notations :

$q = M_{\text{satellite}} / M_{\text{planet}}$ ,  $D = M_{\text{disk}} / M_{\text{planet}}$ ,  $r$  = satellite's orbital radius,  $T_R$  = orbital period at  $r_R$ .  
 $\Delta = (r - r_R) / r_R$ ,  $F$  = the flux of material crossing  $r_R$  (assumed constant),  $\tau_d = M_{\text{disk}} / (F T_R)$ .  
 Due to interactions with the disk, a satellite migrates outwards, at a rate proportionnal to  $qD\Delta^{-3}$ .  
 (Lin & Papaloizou 1979, Goldreich & Tremaine 1980)

## Continuous regime :

The first body forms by gathering all the incoming material :  $M_{\text{satellite}}(t) = F \times t$ .

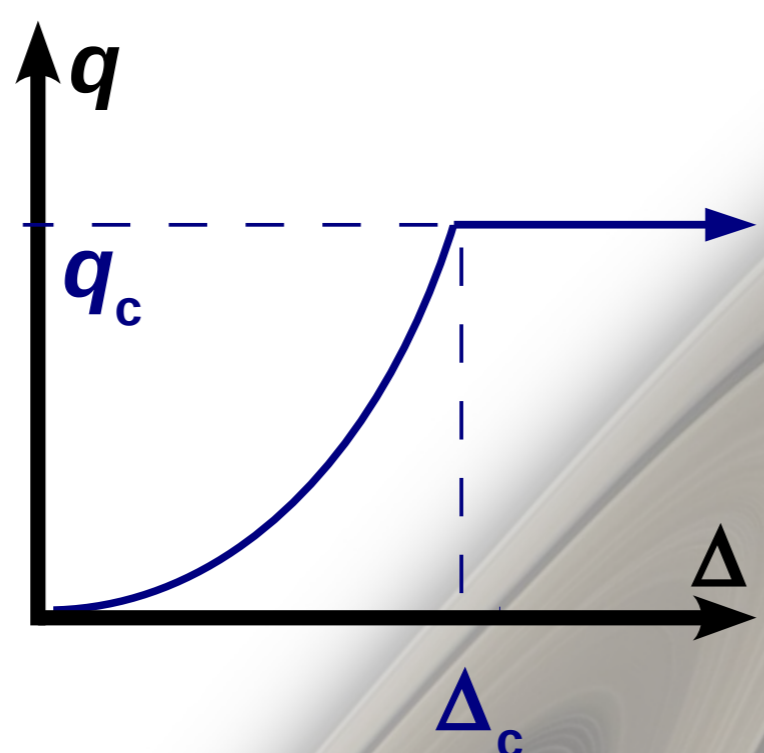
As it migrates,  $q = (3^3/2^6\tau_d)^{1/2} \Delta^2$ .

This stands as long as the satellite is not too far from the disk, i.e. :

$\Delta < 2r_{\text{Hill}}/r$ . This gives :

$$\Delta < \Delta_c = (3 / \tau_d)^{1/2}$$

$$q < q_c = \sim 2 \tau_d^{-3/2}$$



After  $q_c$  or  $\Delta_c$  is reached, the satellite goes on migrating outwards at constant mass, and a new satellite forms.

## Pyramidal regime :

Satellites of mass  $q_c$  are produced every  $\delta t$ , at  $\Delta_c$ , and migrate outwards. As their migration speed decreases, they merge in pairs, producing regularly  $2q_c$  bodies, who merge further, and so on...

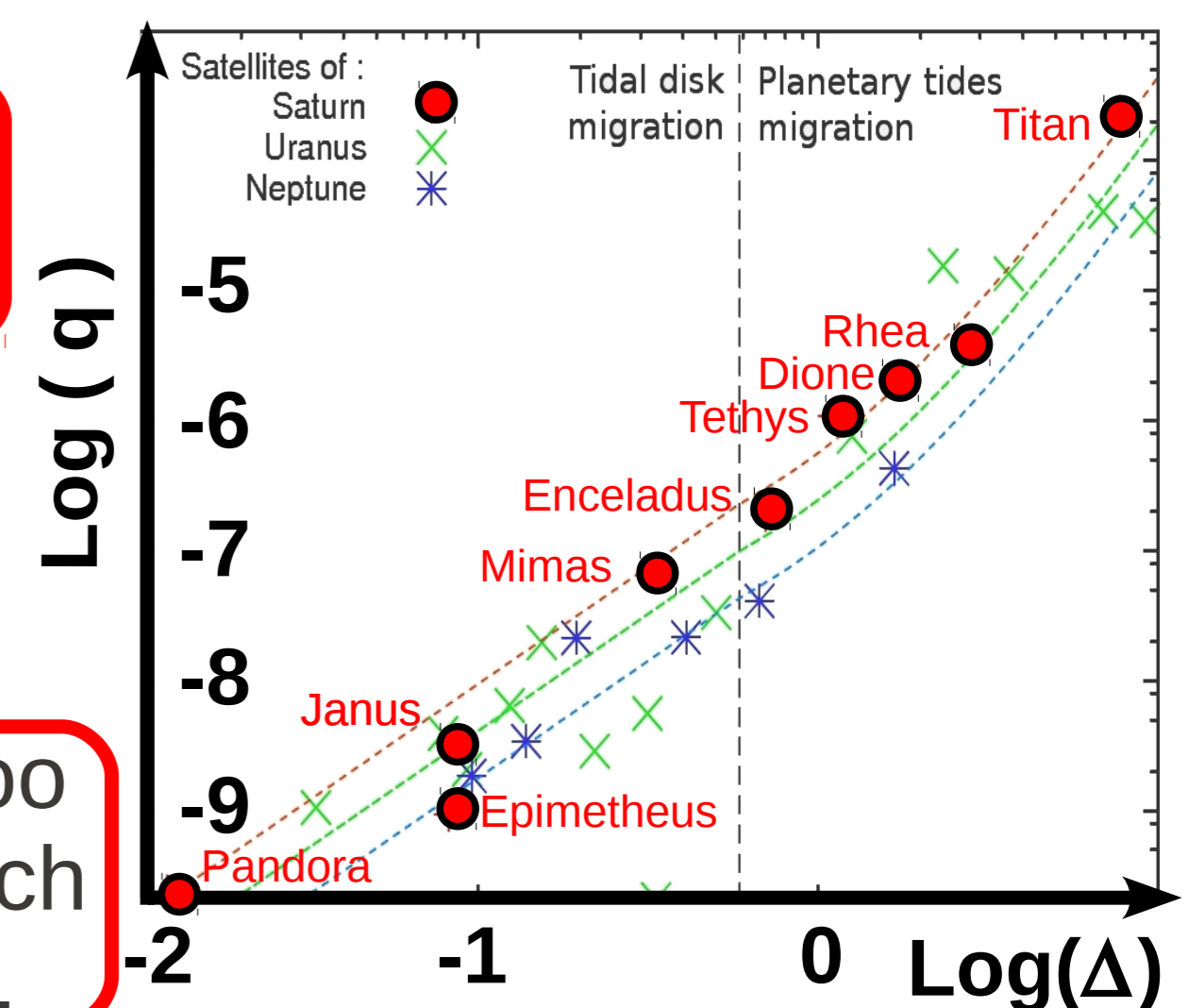
In this process :

$$q \propto \Delta^{9/5} \text{ for } \Delta < 0.58$$

$$q \propto r^{3.9} \text{ for } \Delta > 0.58$$

This matches the Saturnian, Uranian, Neptunian systems.

→ Uranus & Neptune too had big rings from which their satellites formed.



## Application :

For each planet of the Solar System, consider a *Minimum Mass Satellites Tidal Disk*, with  $D=150\%$  times the total mass of present regular satellites.

$D$  is linked to  $\tau_d$  (thus  $\Delta_c$  and  $q_c$ ) via a relation between the disk surface density and its viscosity (Daisaka et al 2001).

✓ **Giant planets :** small  $D$ , large  $\tau_d$ , small  $\Delta_c$ ,  $q_c$  : the pyramidal regime dominates → many satellites of increasing mass starting from the Roche radius.

✓ **Moon forming disk :** large  $D$ , small  $\tau_d$  →  $q_c \sim D$  : continuous regime, the Earth should have only 1 big Moon!

