Protostars and Planets VI
Heidelberg

Small Planetesimals Formed Mars

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- Mars Formation timescale from Hf/W
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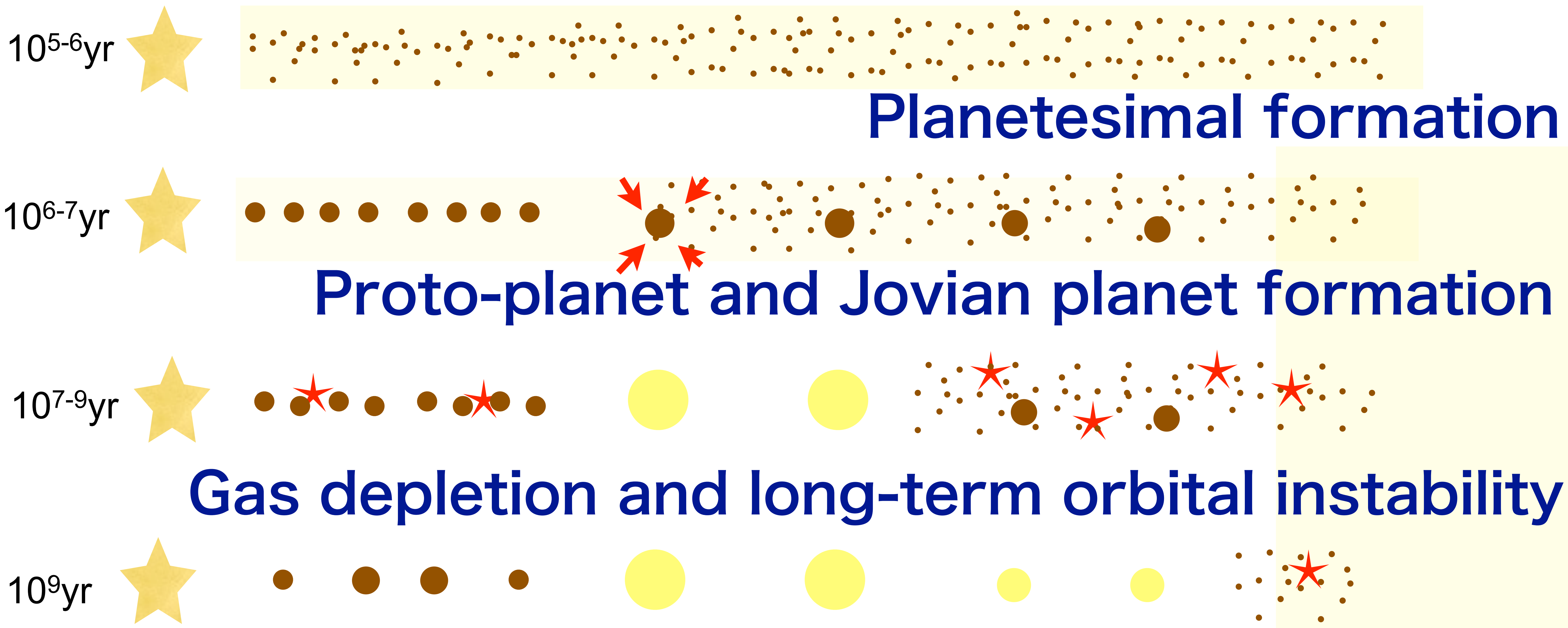
(Kobayashi & Dauphas 2013, Icarus 225, 122-130)

Terrestrial Planet Formation



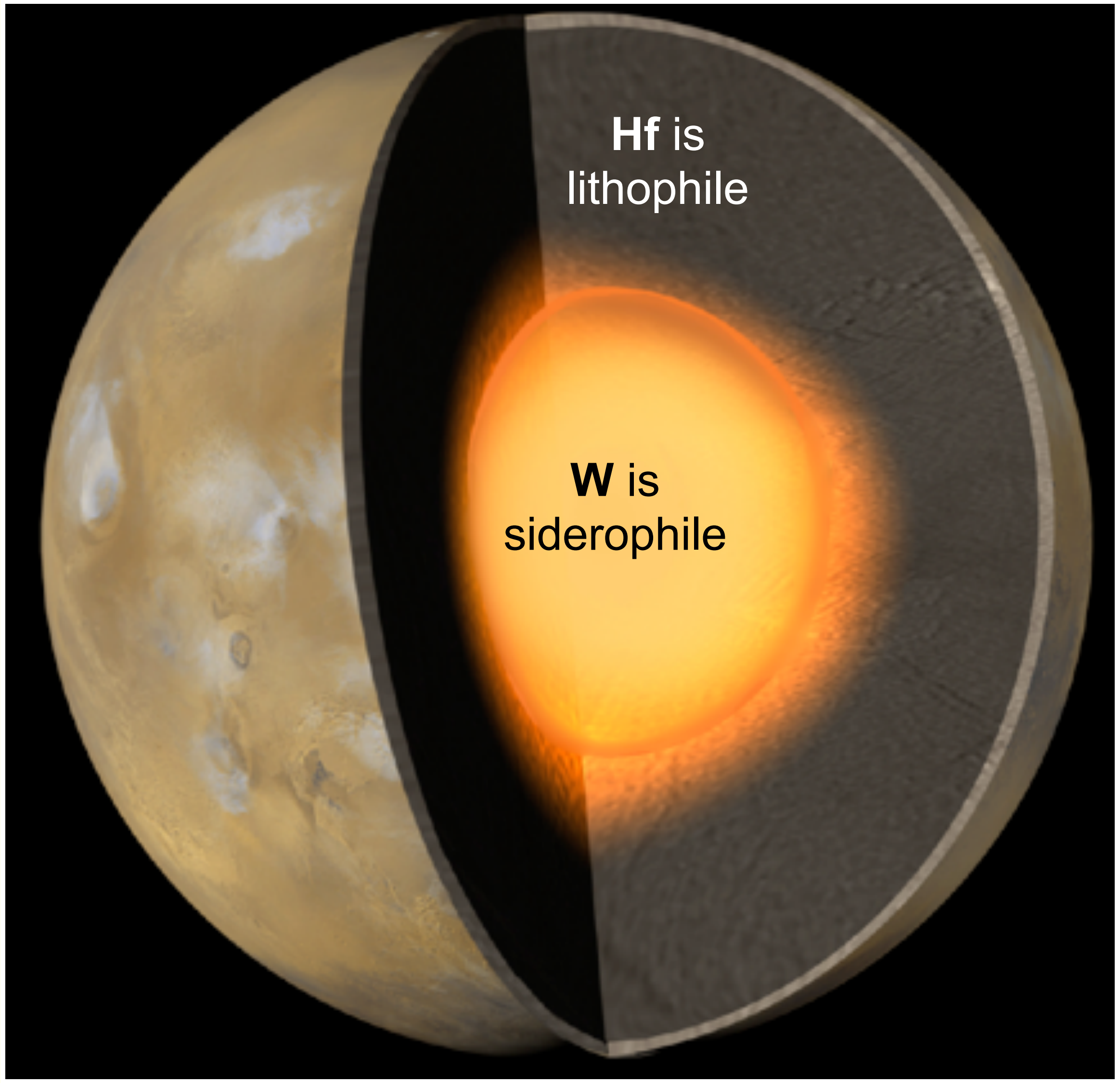
Planetesimal formation

Proto-planet and Jovian planet formation

Gas depletion and long-term orbital instability

- Several tens Mars mass protoplanets formed in a gas disk.
- After gas depletion (~10Myr), collisions among the protoplanets formed Earth and Venus via long-term orbital instability.
- Leftover protoplanets become Mars and Mercury.
  - Indeed, Earth formation age (~50 Myr) is much later than Mars (~4Myr).

Core Formation



Hf: lithophile

retained in mantle.

W: siderophile

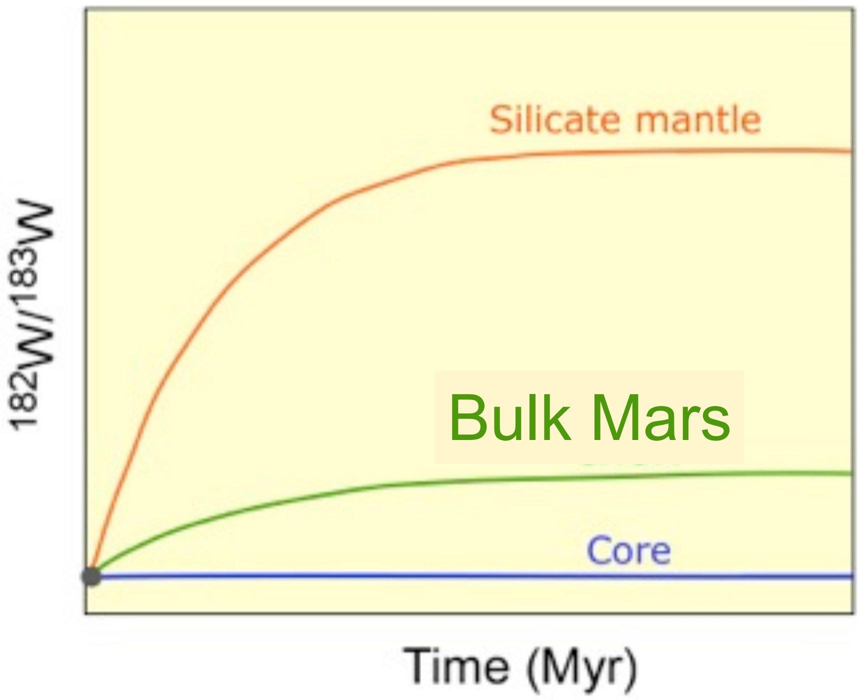
partitioned into core.

$^{182}Hf \rightarrow ^{182}W$

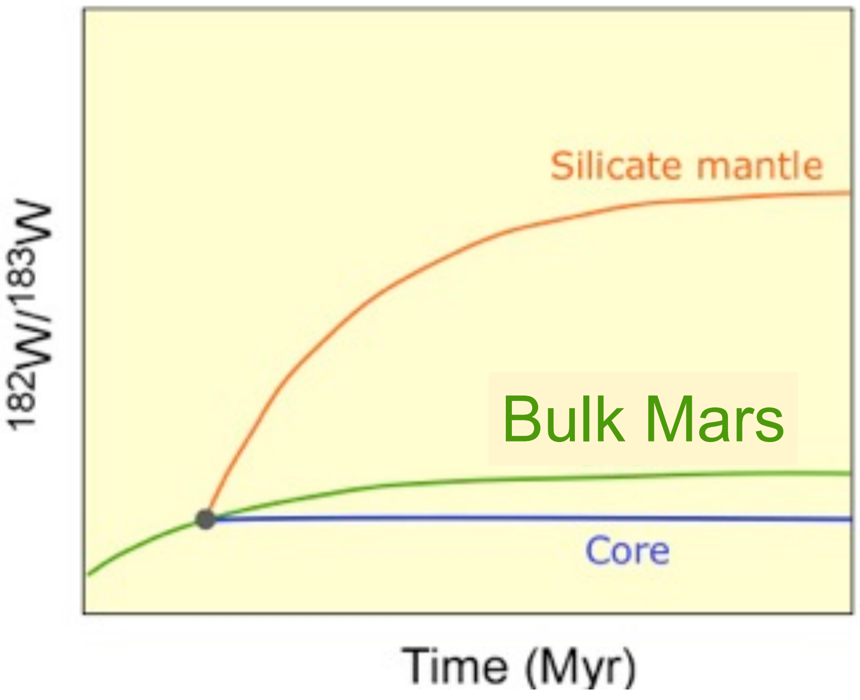
$(t_{1/2} = 9 Myr)$

182W excess in the mantle indicates age.

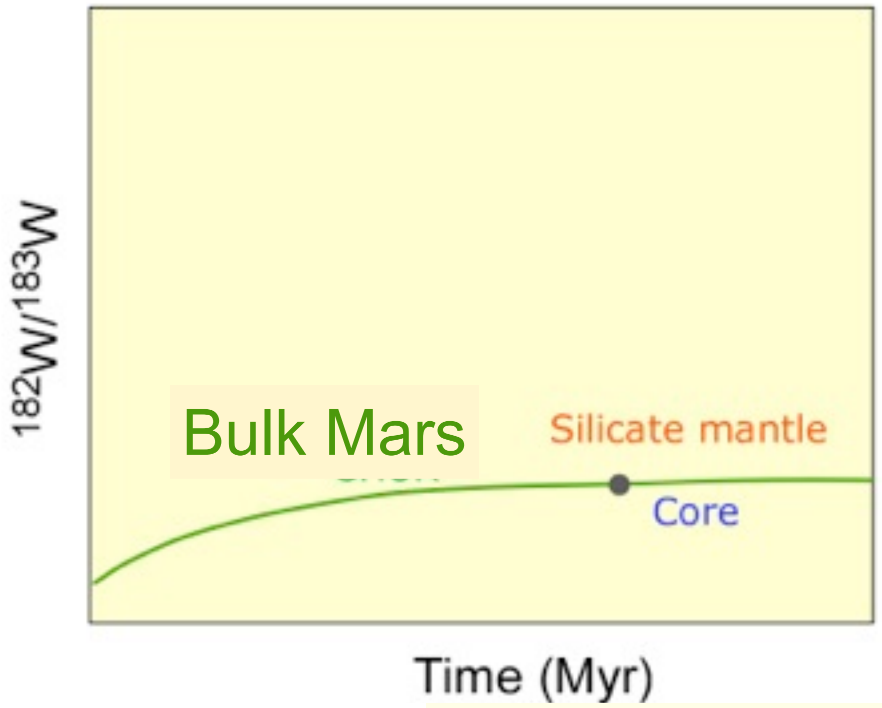
From ^{182}W/^{183}W ratios to time



- Core-mantle differentiation at t=0.



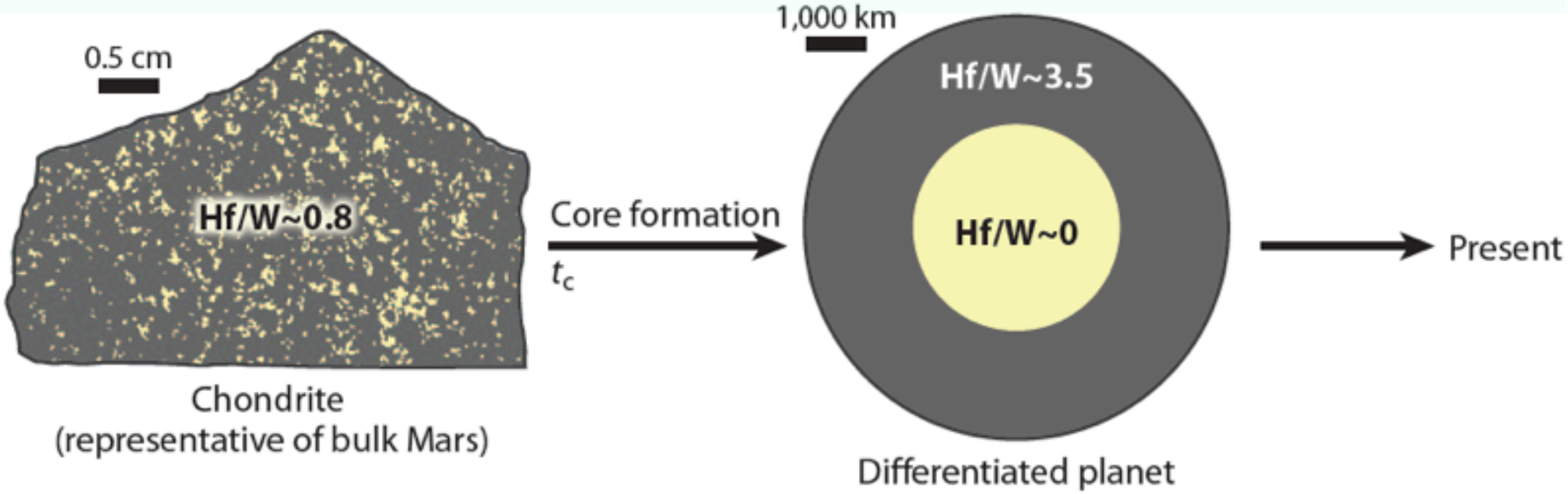
- Core-mantle differentiation while ^{182}Hf is still alive.

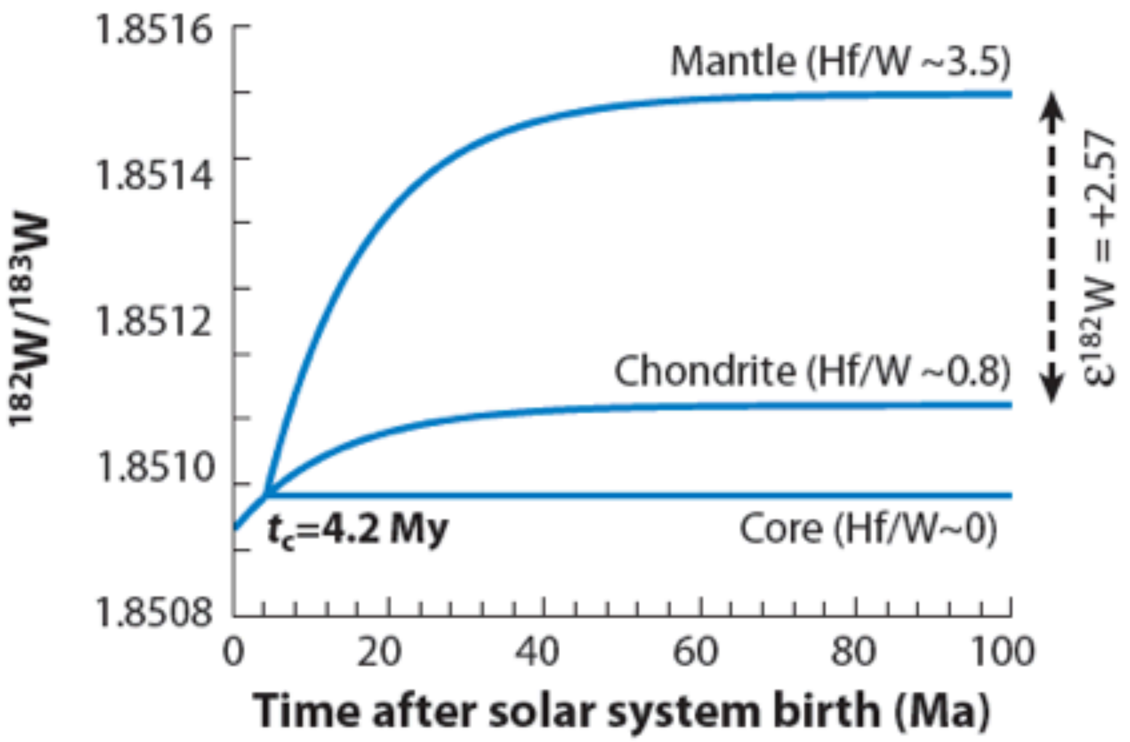


- Core-mantle differentiation after complete decay of ^{182}Hf.

The 182W excess in the martian mantle indicates the core formation age of Mars.

Early Core Formation





The two-stage age of core formation on Mars is ~4.2 Myr

^{182}W excess

change in the mantle

extraterrestrial delivery

removal to the core

radioactive decay

$$\frac{d}{dt}\left(M_{\mathrm{mantle}}\,[^{182}\mathrm{W}]_{\mathrm{mantle}}\right) = [^{182}\mathrm{W}]_{\mathrm{CHUR}}\frac{dM}{dt} - D^{\mathrm{W}}\,[^{182}\mathrm{W}]_{\mathrm{mantle}}\frac{dM_{\mathrm{core}}}{dt} + \lambda\,[^{182}\mathrm{Hf}]_{\mathrm{mantle}}M_{\mathrm{mantle}},$$

$$[^{182}\mathrm{W}]_{\mathrm{mantle}}M_{\mathrm{mantle}} + [^{182}\mathrm{W}]_{\mathrm{core}}M_{\mathrm{core}} = [^{182}\mathrm{W}]_{\mathrm{CHUR}}M_{\mathrm{tot}}$$

$$[^{182}\mathrm{Hf}]_{\mathrm{mantle}}M_{\mathrm{mantle}} = [^{182}\mathrm{Hf}]_{\mathrm{CHUR}}M_{\mathrm{tot}}$$

Assumption: D^{W} & $M_{\mathrm{core}}/M_{\mathrm{mantle}}$ are constant.

W & Mass Relation

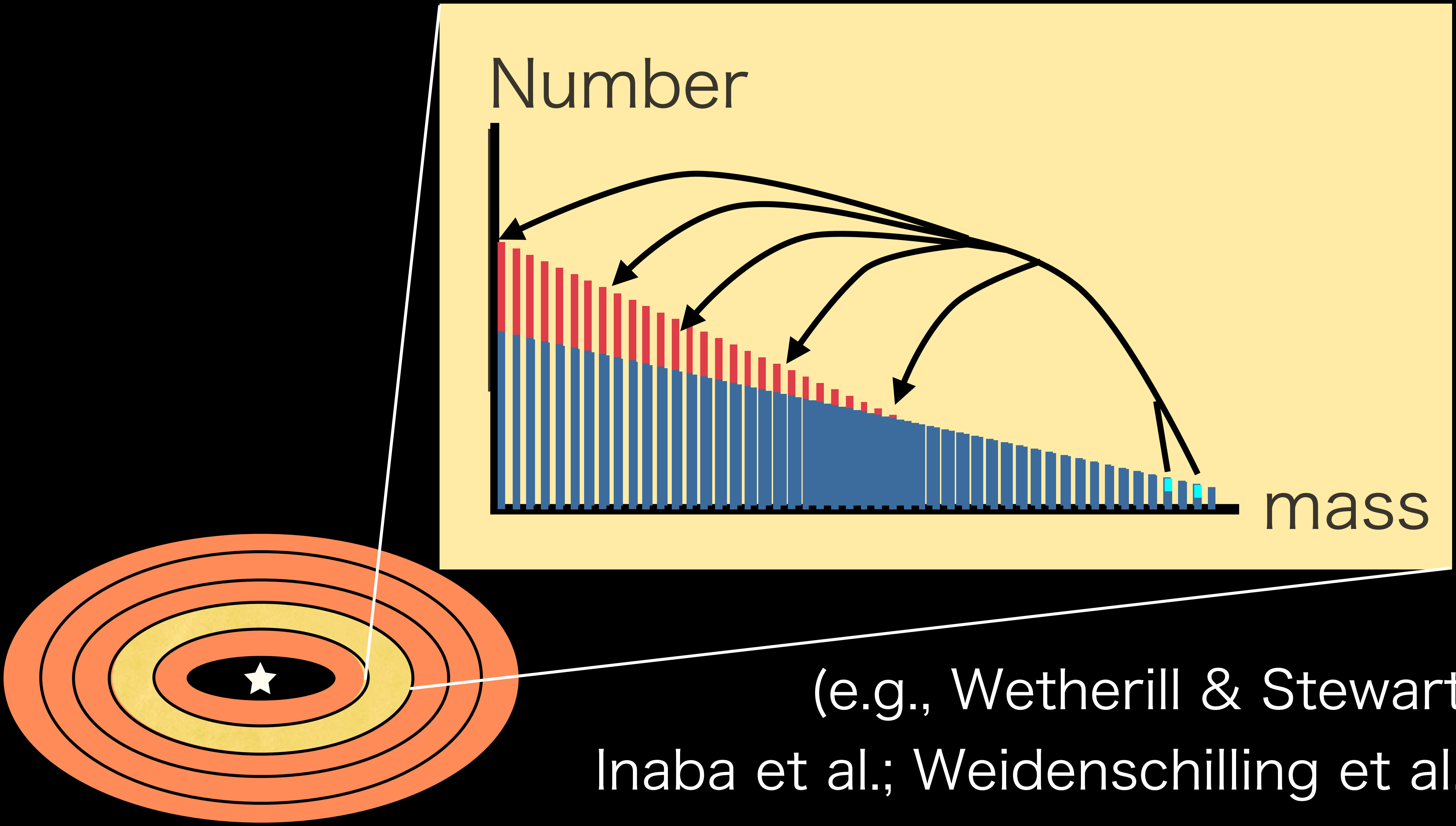
(Jacobson 2005; Dauphas & Pourmand 2011)

$$\begin{aligned}
\varepsilon^{182}\mathrm{W}_{\mathrm{mantle}} &\equiv \left[\frac{\left({}^{182}\mathrm{W}/{}^{183}\mathrm{W}\right)_{\mathrm{mantle}}}{\left({}^{182}\mathrm{W}/{}^{183}\mathrm{W}\right)_{\mathrm{CHUR}}} - 1\right] \times 10^4 \\
&= q_{\mathrm{W}} \left(\frac{{}^{182}\mathrm{Hf}}{{}^{180}\mathrm{Hf}}\right) f^{\mathrm{Hf/W}}_{\mathrm{mantle}} \lambda \\
&\quad \times \int_0^t \left(\frac{M(t')}{M(t)}\right)^{1+f^{\mathrm{Hf/W}}_{\mathrm{mantle}}} \exp(-\lambda t') dt'
\end{aligned}$$

$$\begin{aligned}
\varepsilon^{182}W_{\mathrm{Mars\,mantle}} &= 2.68 \pm 0.19, \\
f^{\mathrm{Hf/W}}_{\mathrm{Mars\,mantle}} &\equiv (\mathrm{Hf/W})_{\mathrm{mantle}}/(\mathrm{Hf/W})_{\mathrm{CHUR}} - 1 = 3.38 \pm 0.56, \\
q_{\mathrm{W}} &\equiv ({}^{180}\mathrm{Hf}/{}^{182}\mathrm{W})_{\mathrm{CHUR}} \times 10^4 = 1.07 \times 10^4, \\
({}^{182}\mathrm{Hf}/{}^{180}\mathrm{Hf}) &= (9.72 \pm 0.44) \times 10^{-5} \\
\lambda &= 0.0779 \pm 0.0008 \mathrm{Myr}^{-1}
\end{aligned}$$

(Dauphas & Pourmand 2011 and references therein)

Statistical Simulation



(e.g., Wetherill & Stewart;
Inaba et al.; Weidenschilling et al.;
Kenyon & Bromley)

Basic Equations

(see HK, Tanaka, Krivov, Inaba 2010)

$$\begin{aligned}
\frac{\partial m n_{\rm s}(m,a)}{\partial t} = {} & \frac{m}{2}\Omega_{\rm K}\int_0^m dm_1 \int_{m-m_1-m_{\rm e}}^{\infty} dm_2 \\
& \times (h_{m_1,m_2}a)^2 n_{\rm s}(m_1,a) n_{\rm s}(m_2,a)\langle P_{\rm col}\rangle \\
& \times \delta(m-m_1-m_2+m_{\rm e}) \\
& -\Omega_{\rm K} m n_{\rm s}(m)\int_0^\infty dm_2 (h_{m,m_2}a)^2 n_{\rm s}(m_2,a)\langle P_{\rm col}\rangle \\
& +\frac{\partial}{\partial m}\Omega_{\rm K}\int_m^\infty dm_1 \int_0^{m_1} dm_2 (m_1+m_2) f(m,m_1,m_2) \\
& \times n_{\rm s}(m_1,a) n_{\rm s}(m_2,a)(h_{m_1,m_2}a)^2\langle P_{\rm col}\rangle \\
& -\frac{1}{a}\frac{\partial}{\partial a}[a m n_{\rm s}(m,a) v_{\rm drift}(m,a)],
\end{aligned}$$

$$\frac{m_{\rm e}}{m_1+m_2} = \frac{\phi}{1+\phi}$$

$$\phi = m_1 m_2 v^2/2(m_1+m_2)^2 Q_{\rm D}^*$$

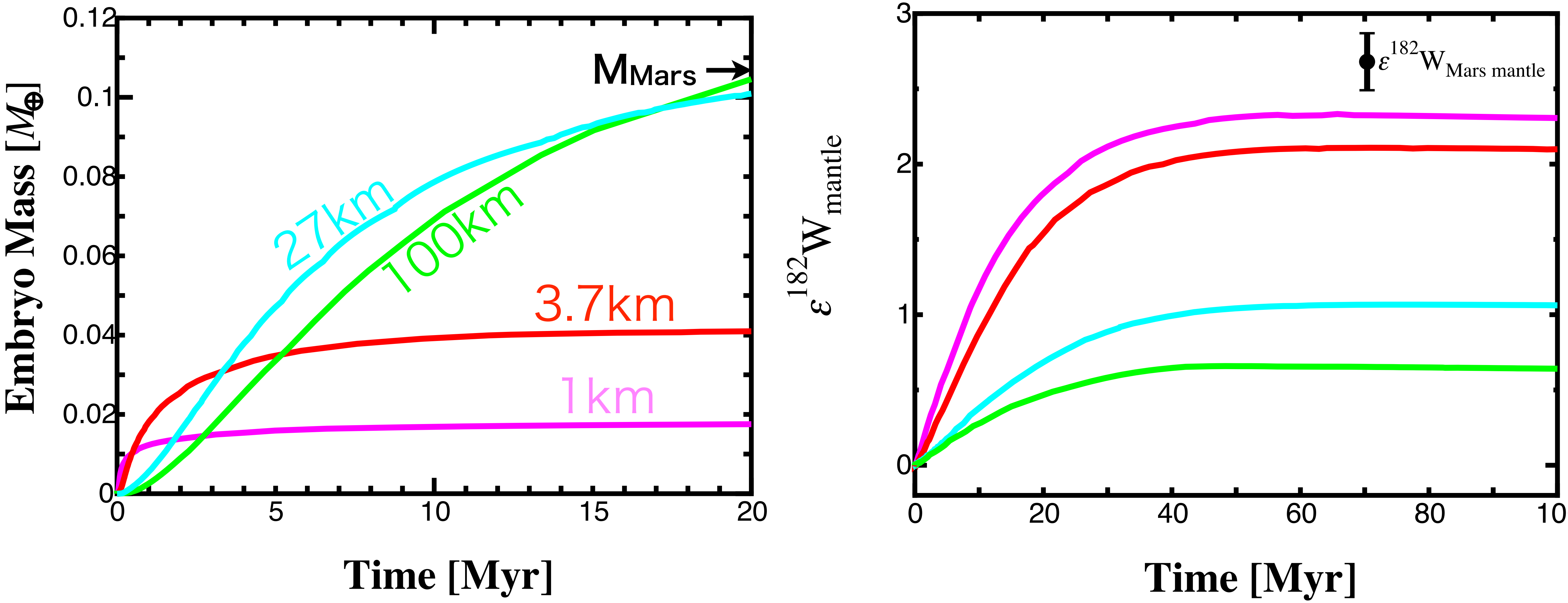
$$\frac{de^{*2}}{dt} = \left(\frac{de^{*2}}{dt}\right)_{\rm grav} + \left(\frac{de^{*2}}{dt}\right)_{\rm gas} + \left(\frac{de^{*2}}{dt}\right)_{\rm coll},$$

$$\frac{di^{*2}}{dt} = \left(\frac{di^{*2}}{dt}\right)_{\rm grav} + \left(\frac{di^{*2}}{dt}\right)_{\rm gas} + \left(\frac{di^{*2}}{dt}\right)_{\rm coll},$$

$$\begin{aligned}
& (m_1+m_2) f(m,m_1,m_2) \\
& = \begin{cases} m_{\rm e}\left(\dfrac{m}{m_{\rm L}}\right)^{-b} & \text{for} \quad m < m_{\rm L}, \\ m_{\rm e} & \text{for} \quad m \geq m_{\rm L}, \end{cases}
\end{aligned}$$

W evolution

In an MMSN disk



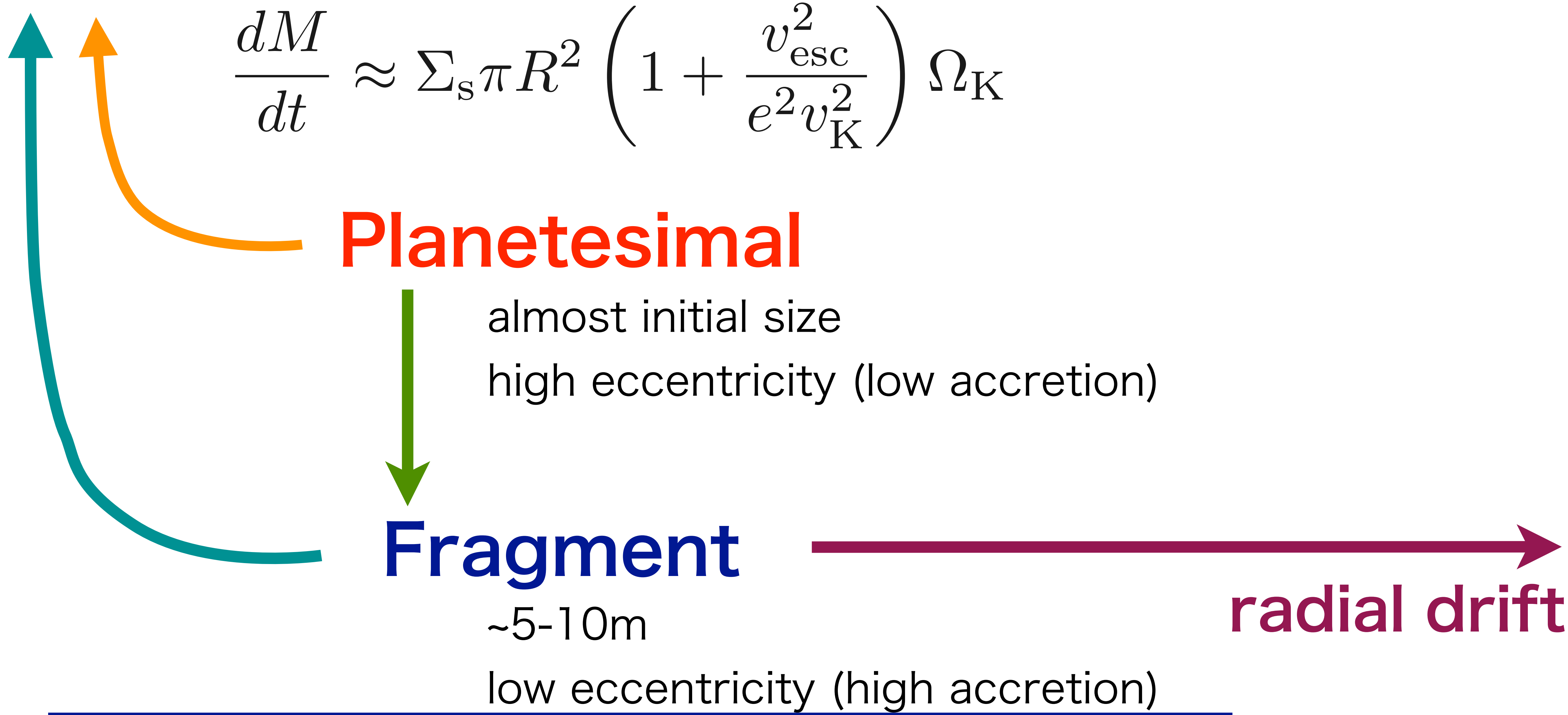
In MMSN, large planetesimals can produce Mars size bodies.

However, W excess is too small.

Embryo Growth

Planetary Embryo (see Ormel & Kobayashi 2012)

$$\frac{dM}{dt} \approx \Sigma_\mathrm{s} \pi R^2 \left(1 + \frac{v_\mathrm{esc}^2}{e^2 v_\mathrm{K}^2}\right) \Omega_\mathrm{K}$$



Planetesimal

almost initial size

high eccentricity (low accretion)

Fragment

~5-10m

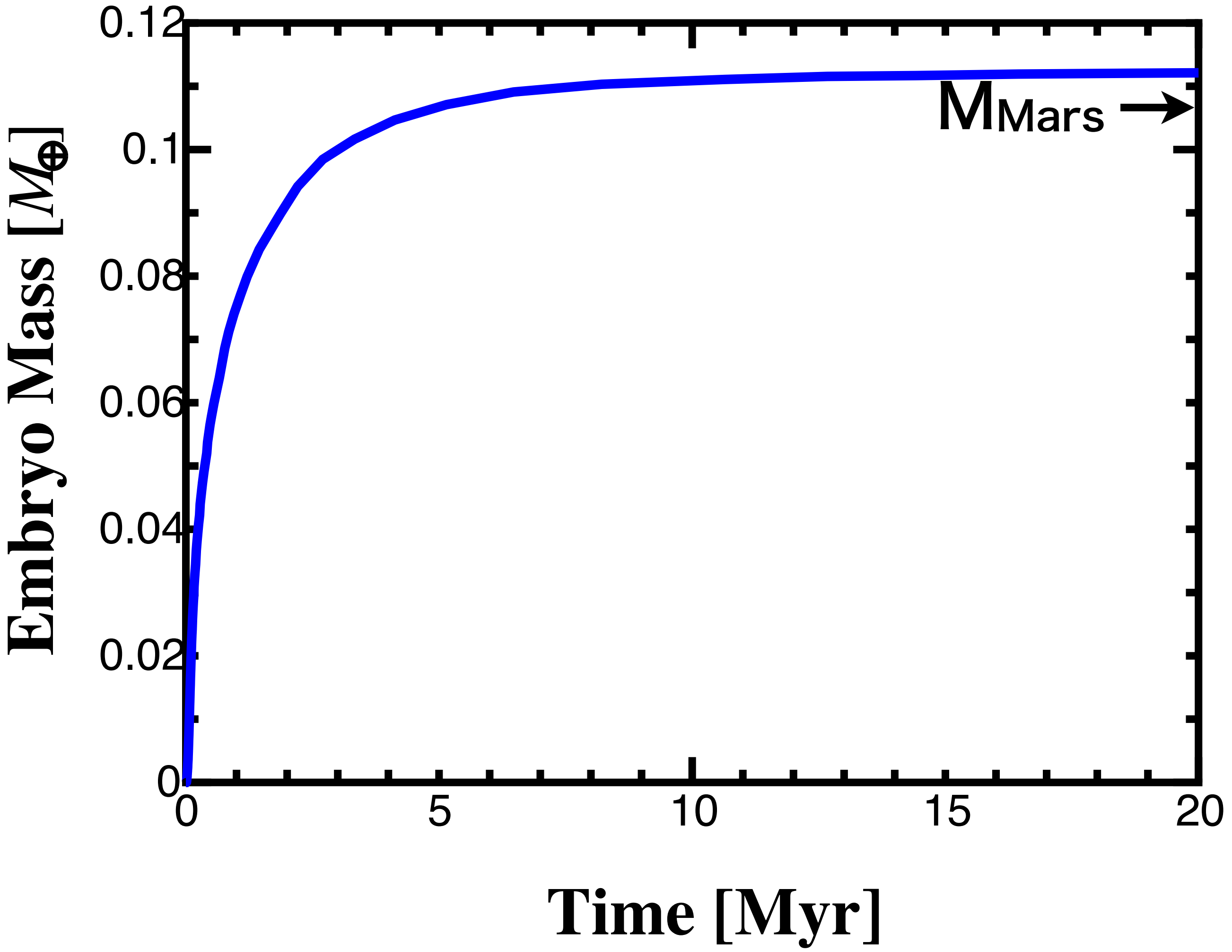
low eccentricity (high accretion)

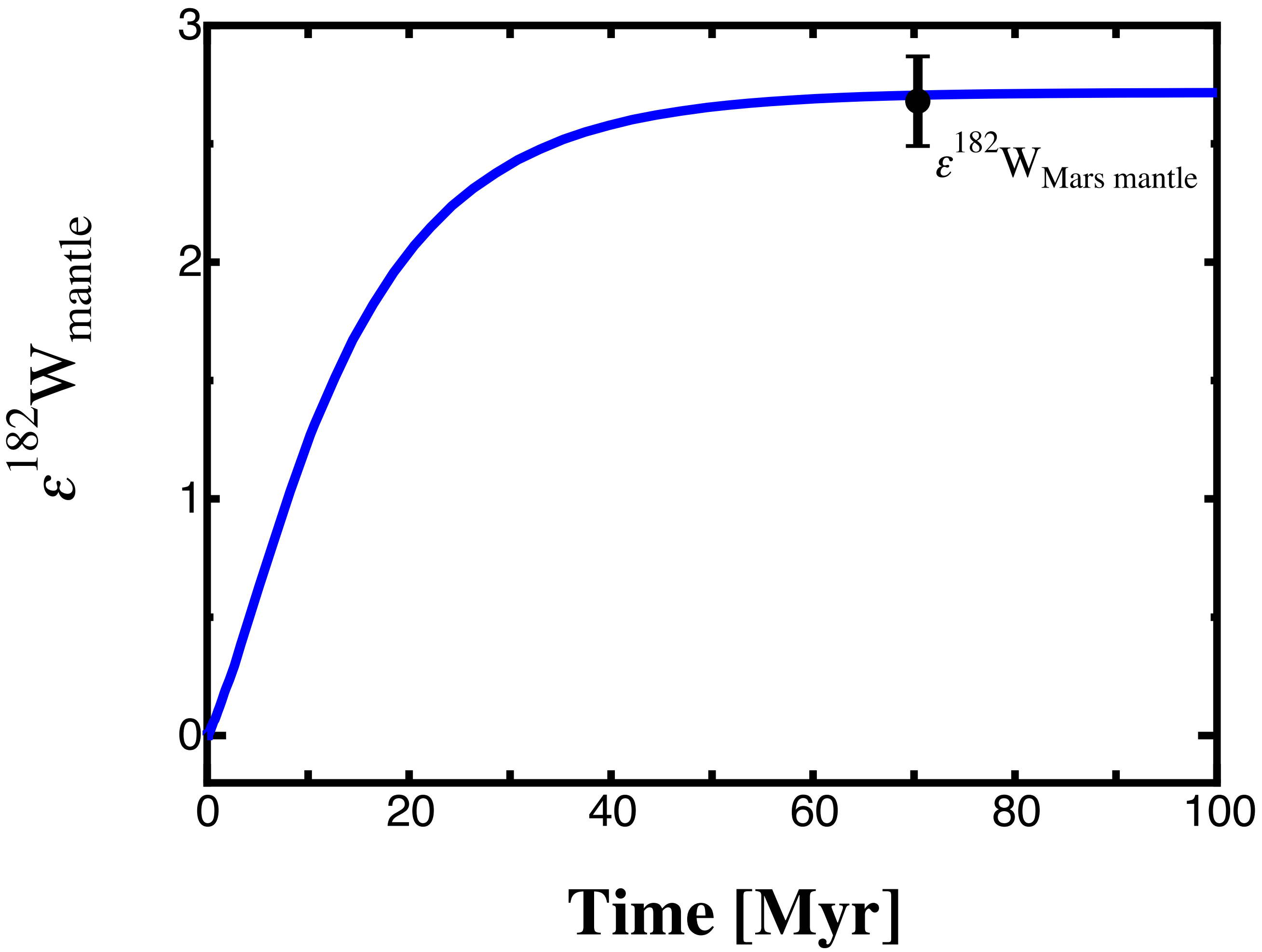
radial drift

Small planetesimals rapidly form embryos, but they are small (and vice versa).

Mass and ε^{182}W

2.7 x MMSN r_0 = 3.7km

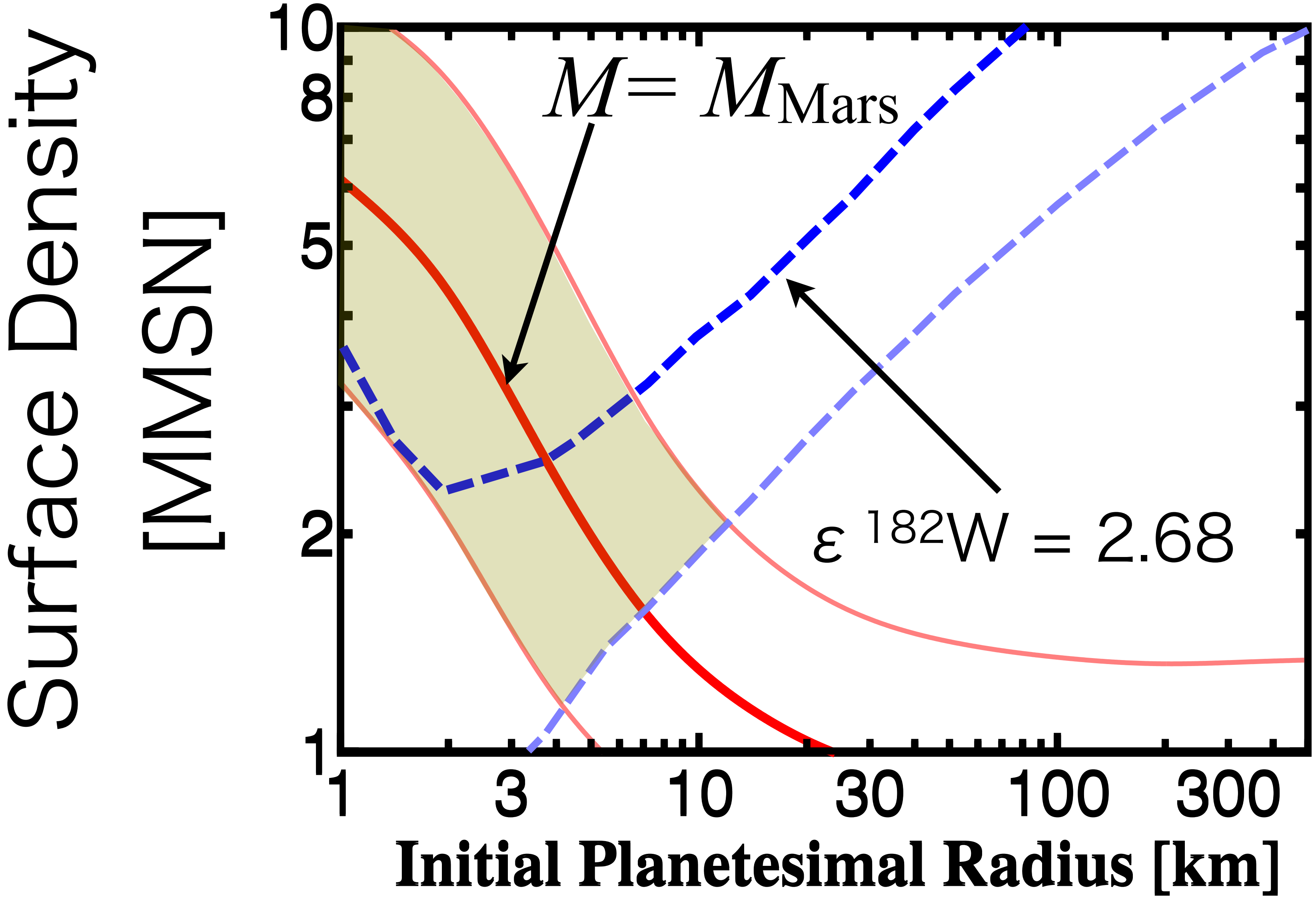




We can find a parameter set to satisfy mass & ε^{182}W.

Mars Condition

88 runs (8x11 for 1-10xMMSN & r_0= 1-500km)

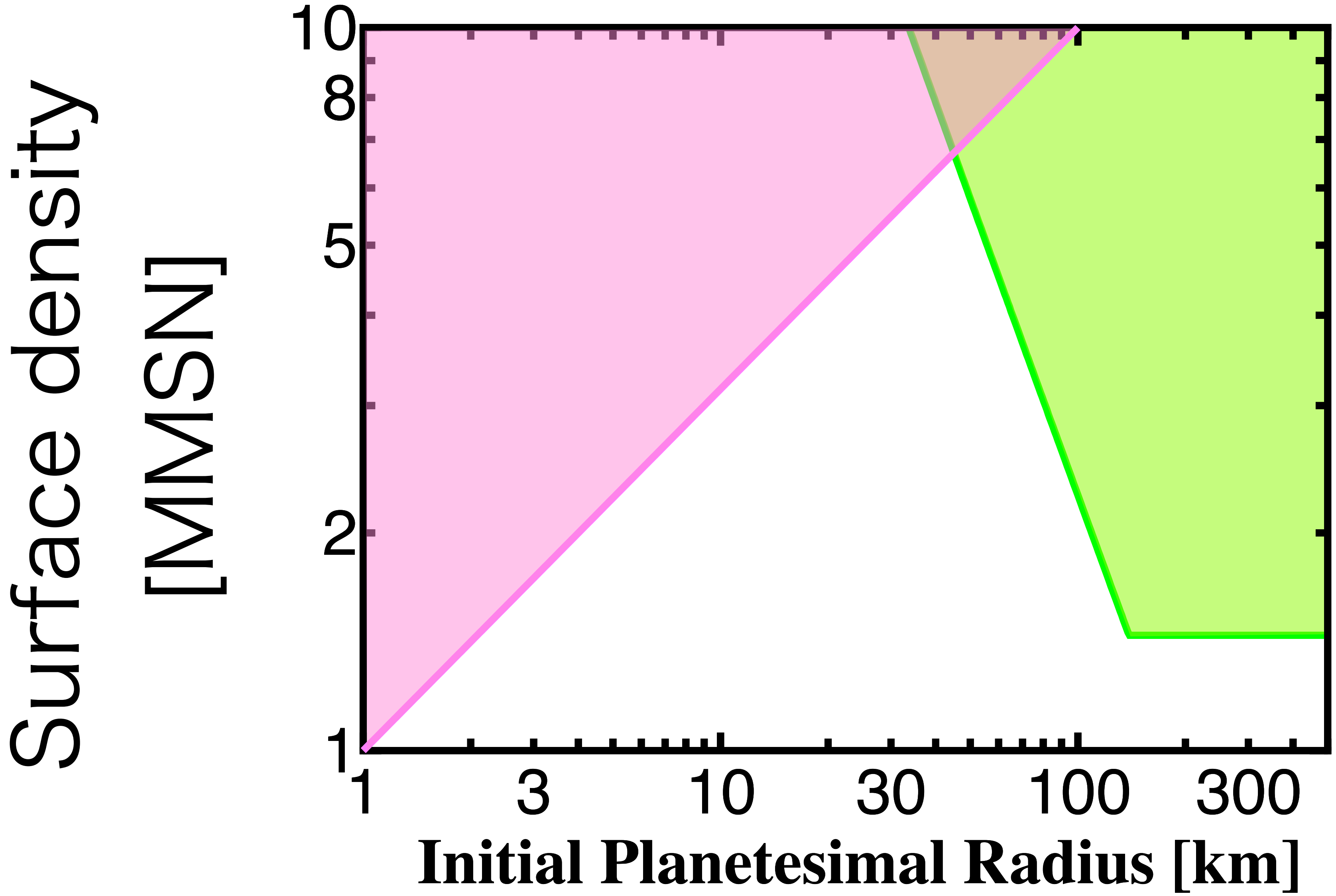


Jupiter & Saturn

To produce the core with ~ 5M⊕ of Jupiter (Kobayashi et al. 2011)

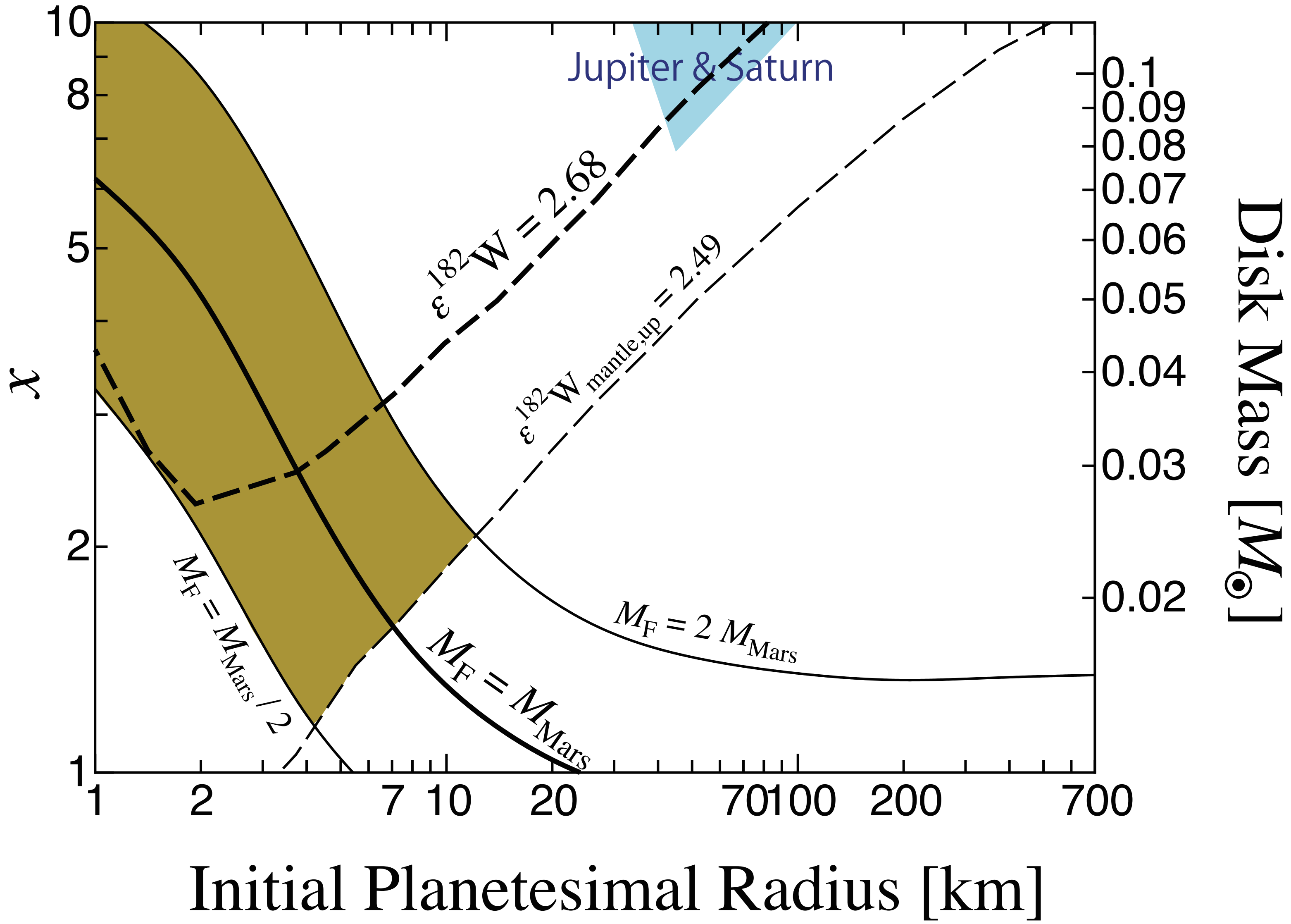
For rapid formation of Saturn's core (Kobayashi et al. 2012)

$$\Sigma = x\Sigma_{\mathrm{MMSN,1AU}}(a/1\mathrm{AU})^{-1.5}$$



Mars, Jupiter & Saturn

$$\Sigma = x\Sigma_{\mathrm{MMSN,1AU}}(a/1\mathrm{AU})^{-1.5}$$



Necessary conditions are different.

Conclusion & Discussion

- The mass of Mars and the W excess in the Martian meteoroids require <10km planetesimals in a relatively massive disk.
- Jupiter & Saturn needs planetesimals with radii 50-100km.
- Different formation mechanisms between icy and rocky planetesimals (e.g., Okuzumi et al. 2012).