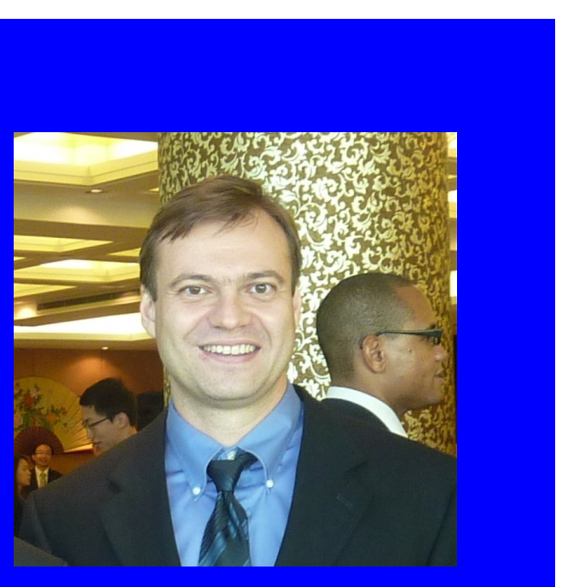
Disk-Planet Interaction: New Results and Implications for Gap Opening

Roman Rafikov (Princeton)
with Cristobal Petrovich, Ruobing Dong, Jim Stone



Roman Rafikov

Gravitational disk-planet coupling has been a subject of intense investigation for more than three decades. Our understanding of very important phenomena such as planet migration and gap opening crucially relies on our knowledge of disk-planet interaction. Despite that, some interesting new results have emerged in this field recently. Some of them are reviewed.

Nonlinear evolution of density waves

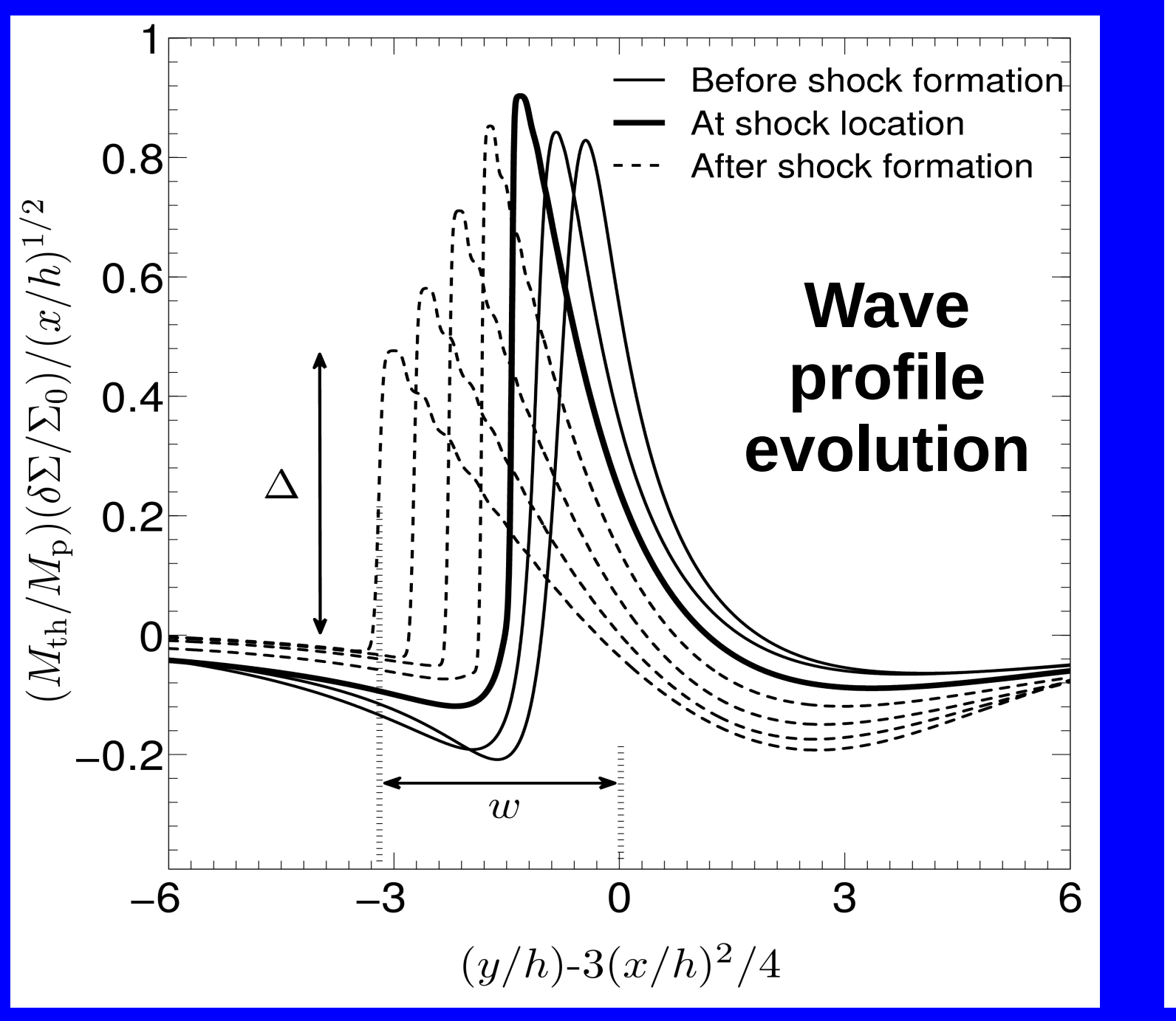
Goodman and Rafikov (2001) demonstrated that density waves launched by even low mass planets in protoplanetary disks evolve nonlinearly and dissipate in weak shocks depositing their angular momentum in the disk. This causes evolution of the disk surface density and can result in gap formation (Rafikov 2002) even for planets less massive than the so-called "thermal mass"

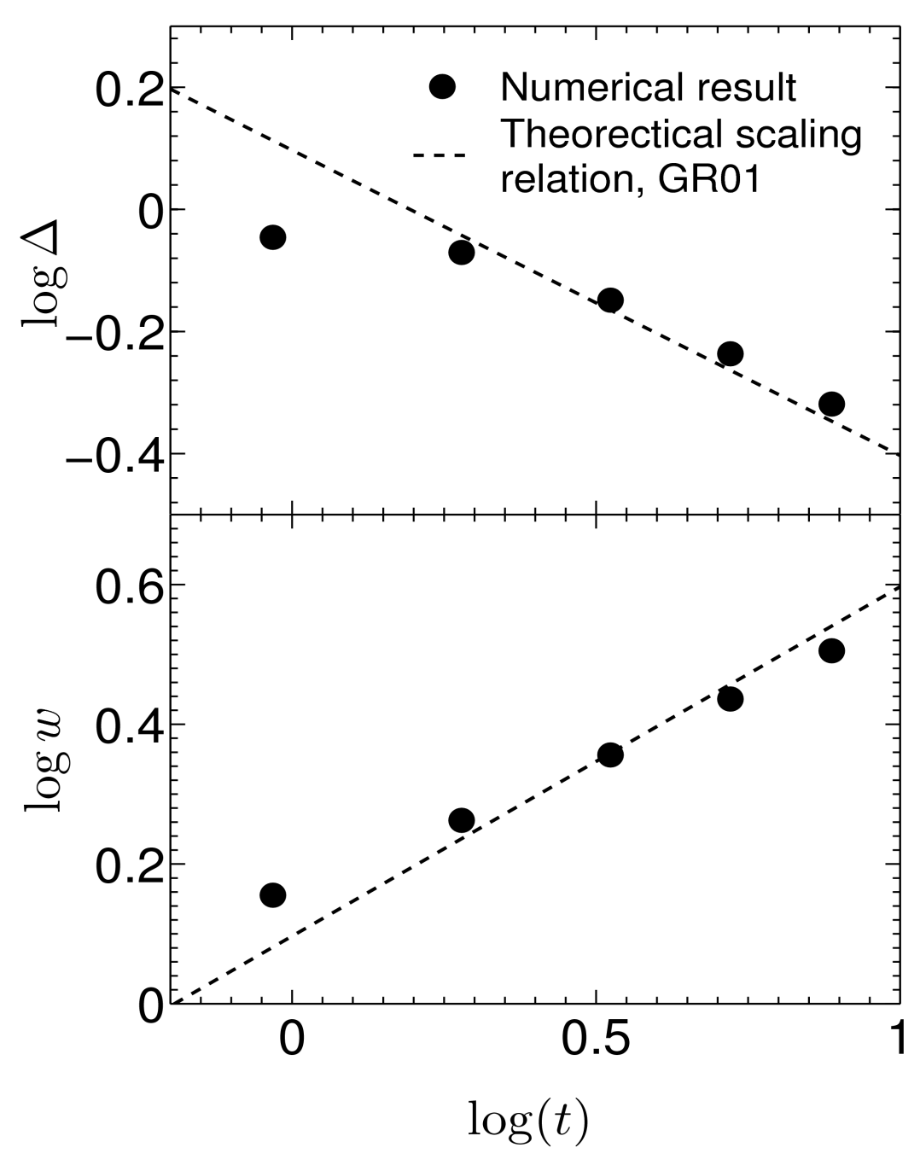
$$M_{\rm th}=\frac{c_s^3}{\Omega G}\qquad M_{\rm th}\approx 12\left(\frac{c}{1\ {\rm km\ s^{-1}}}\right)^3\left(\frac{r_p}{1\ {\rm AU}}\right)^{3/4}M_\oplus$$

Nonlinear wave evolution has been recently confirmed numerically using Godunov code Athena in Dong et al (2012a,b). Analytical scalings have been verified with high accuracy.

Nonlinear steepening, followed by the decay and broadening of the density wave profile, compared to theoretical predictions

$$\Delta\propto t^{-1/2},\qquad w\propto t^{1/2}$$



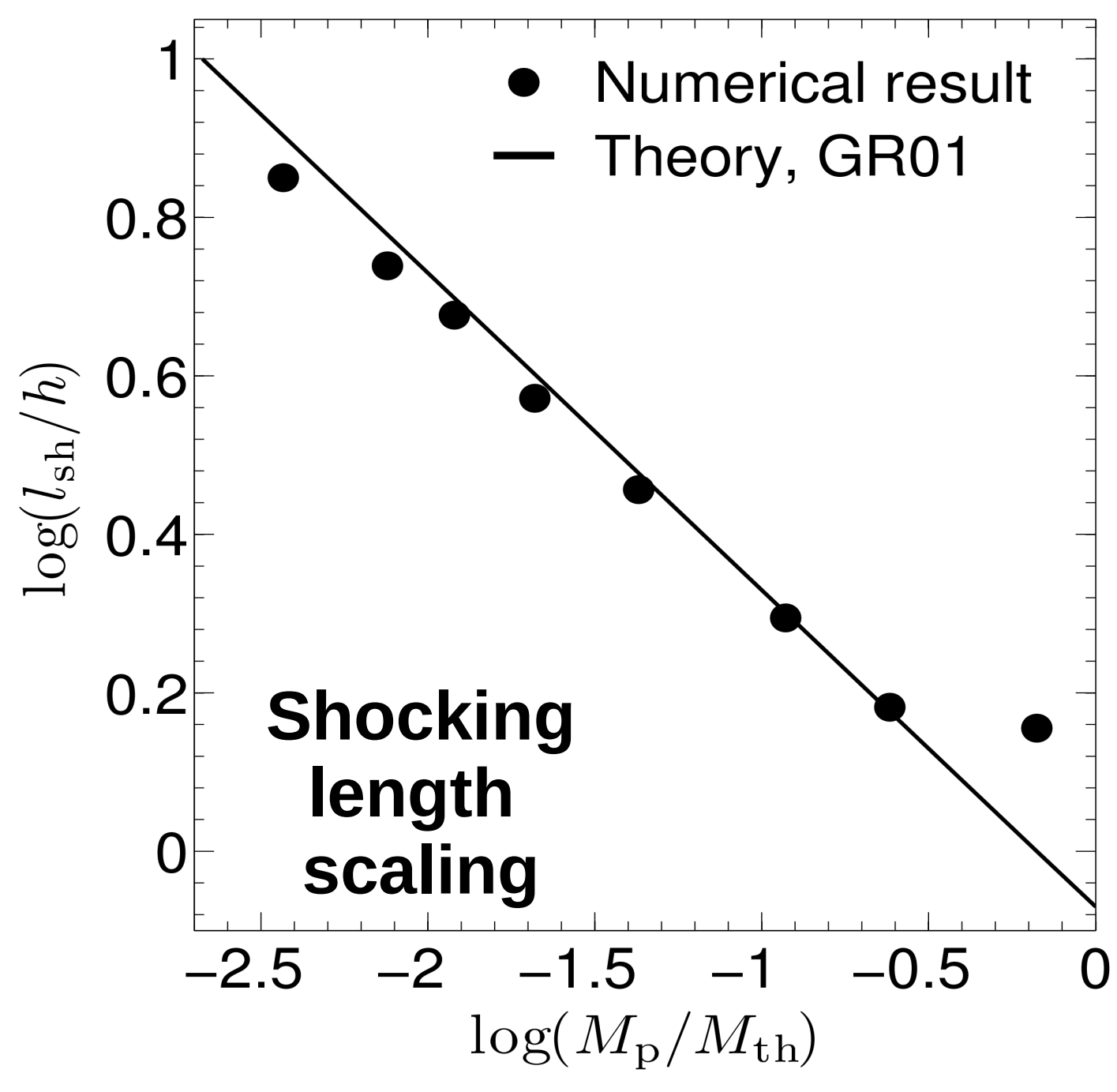


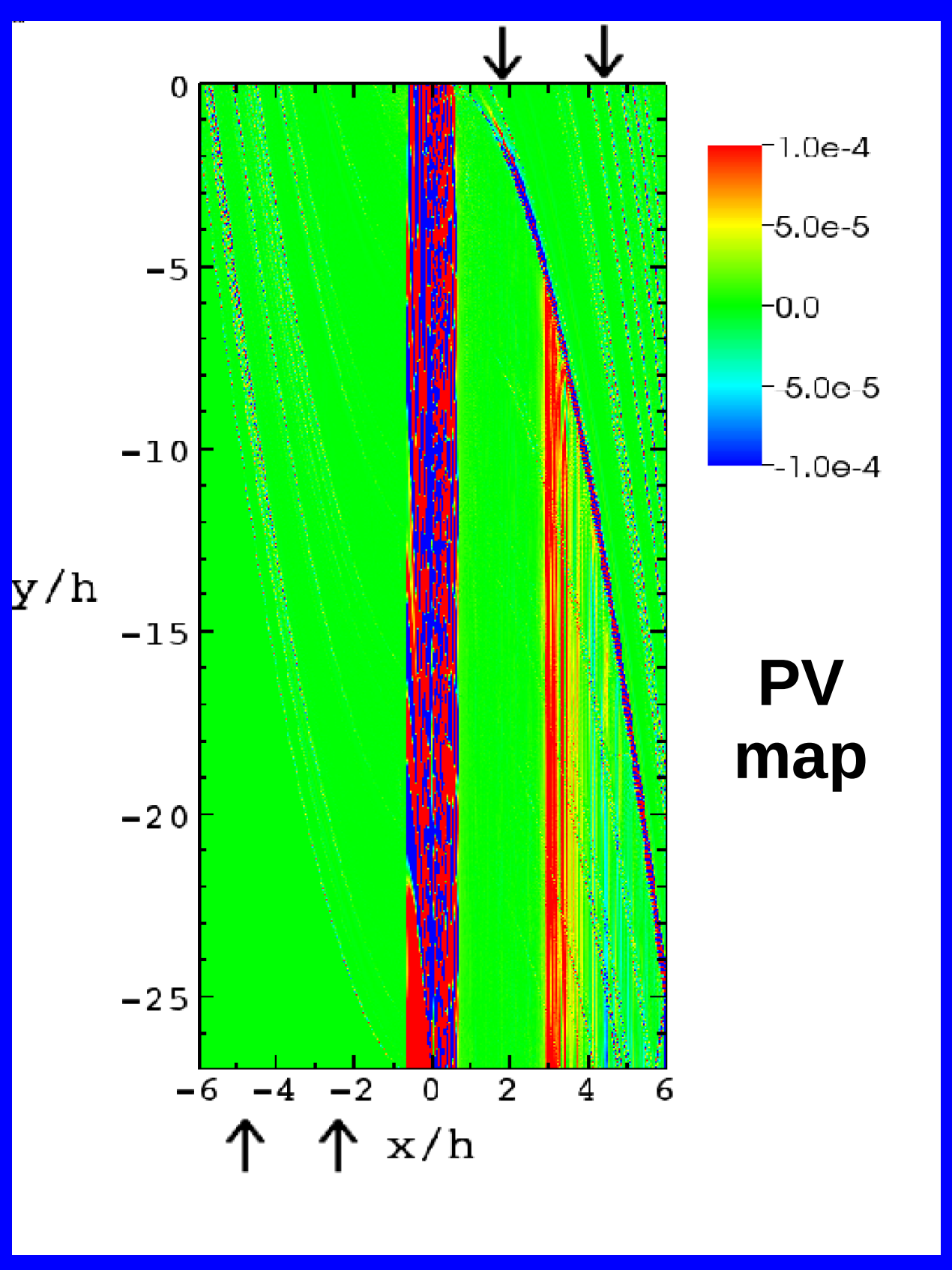
Shocking distance

$$l_{\rm sh}\approx 0.8\left(\frac{\gamma+1}{12/5}\frac{M_{\rm p}}{M_{\rm th}}\right)^{-2/5}h$$

Radial distance away from the planet at which density wave shocks. Very important quantity telling us where the angular momentum of the wave gets deposited into the disk. Surface density of the disk can evolve (and gap may form) only beyond the shocking distance! In the absence of viscosity only dissipation at the shock can damp the wave. Analytical scaling was verified to a good degree, including very low mass planets (equivalent of 3 Lunar masses at 1 AU) and very small density contrasts in the wave (0.003).

The fact that density waves launched even by low mass planets can efficiently dissipate beyond several disk scale heights means that disk surface density can be perturbed even by relatively small perturbers. This implies in particular, that gaps can be opened not only by massive, Jupiter-size planets, but also by lower mass (several to several tens of Earth mass) objects. This can work if (1) the disk viscosity is low and (2) planet is high enough in mass to not migrate out of the forming gap. When these conditions are fulfilled global simulations demonstrate gap opening by low mass planets (Duffell & MacFadyen 2013). This has important implications for stopping Type I migration of super-Earths and Neptunes close to their stars. There the thermal mass is low and planets should be able to open gaps, slowing down their migration. This is a potential parking mechanism for close-in planets.

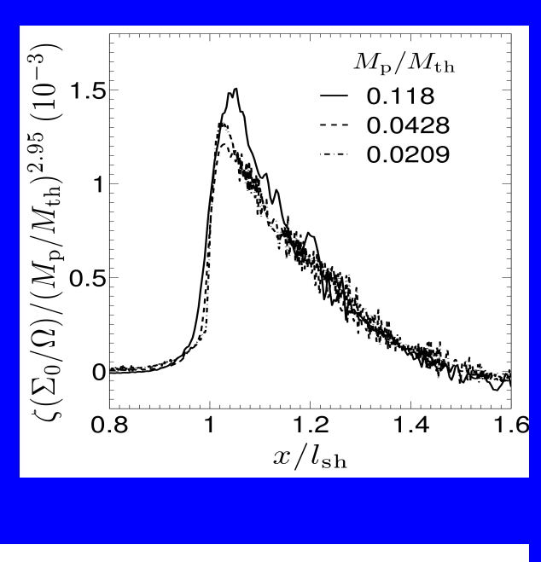


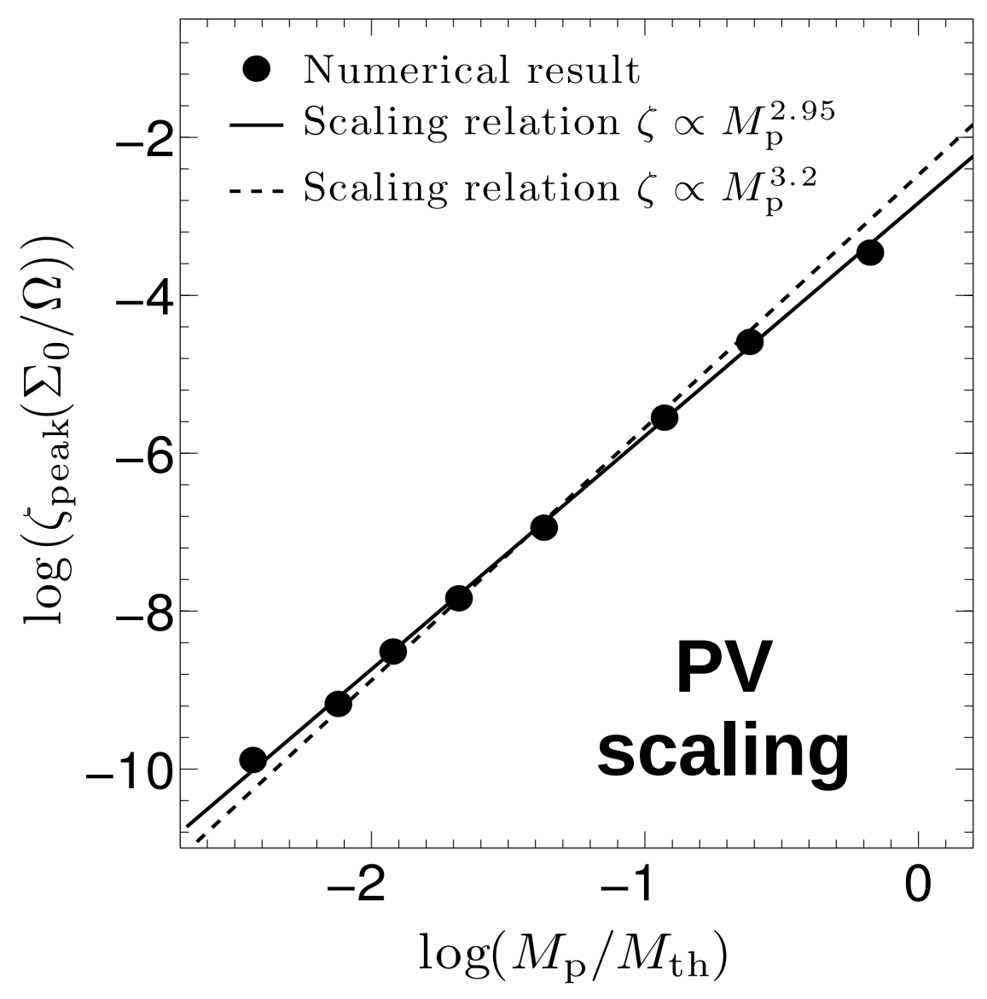


Potential vorticity (PV)

$$\zeta\equiv\frac{\mathbf{e}_z\cdot(\nabla\times\mathbf{v})+2\Omega}{\Sigma}$$

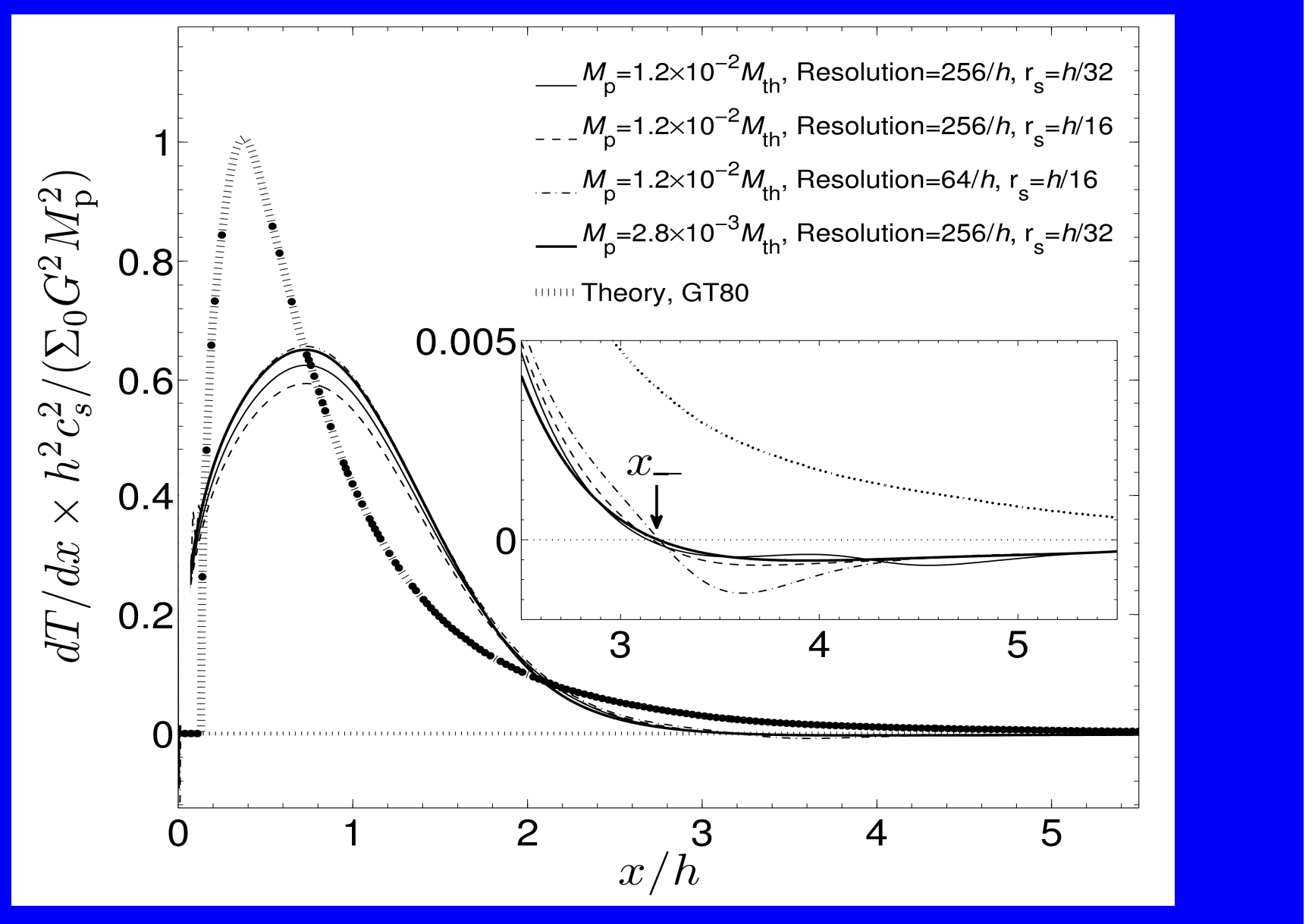
PV is another useful quantity that can be used both to verify theory and for diagnostic purposes. Without explicit viscosity PV is conserved. It is excited at the shock and its emergence in the simulations has been used to determine the shocking distance. PV can be computed analytically in the weak shock approximation. In particular one can show the the amplitude of PV excited at the shock scales with the planet mass to the power of 16/5. Again, our simulations show good agreement with this theoretical scaling. Also, the rescaled shape of the PV profile is universal, i.e. independent of the planet mass.





Negative torque density phenomenon

The way in which planet affects the disk depends on the radial distribution of the angular momentum deposited in the disk by the density wave. The latter results from a convolution of two processes: (1) wave excitation and (2) wave dissipation (covered on the left). Wave excitation was first calculated by Goldreich & Tremaine (1980) who showed that the excitation torque density (the amount of angular momentum excited by the planet per unit time and radial distance) in a homogeneous disk scales as the inverse 4-th power of the radial separation from the planet, see formula below. This simple prescription has been used extensively to explore gap formation both in protoplanetary disks and in disks around supermassive black hole binaries in centers of galaxies.
Our recent numerical work (Dong et al 2011a) shows this picture to be incomplete – simulations show the torque density to change sign at 3.2 h away from the planet. Rafikov & Petrovich (2012) revised the linear calculation of the excitation torque density by carrying it out in physical space and explained this effect.



Classical GT80 asymptotic torque density

$$\frac{dT}{dr}\to{\rm sgn}(r-r_p)C_{\rm GT80}\frac{(GM_p)^2\Sigma_0}{\Omega^2|r-r_p|^4},$$
$$C_{\rm GT80}=\frac{32}{81}\left[2K_0\left(\frac{2}{3}\right)+K_1\left(\frac{2}{3}\right)\right]^2\approx 2.50783.$$

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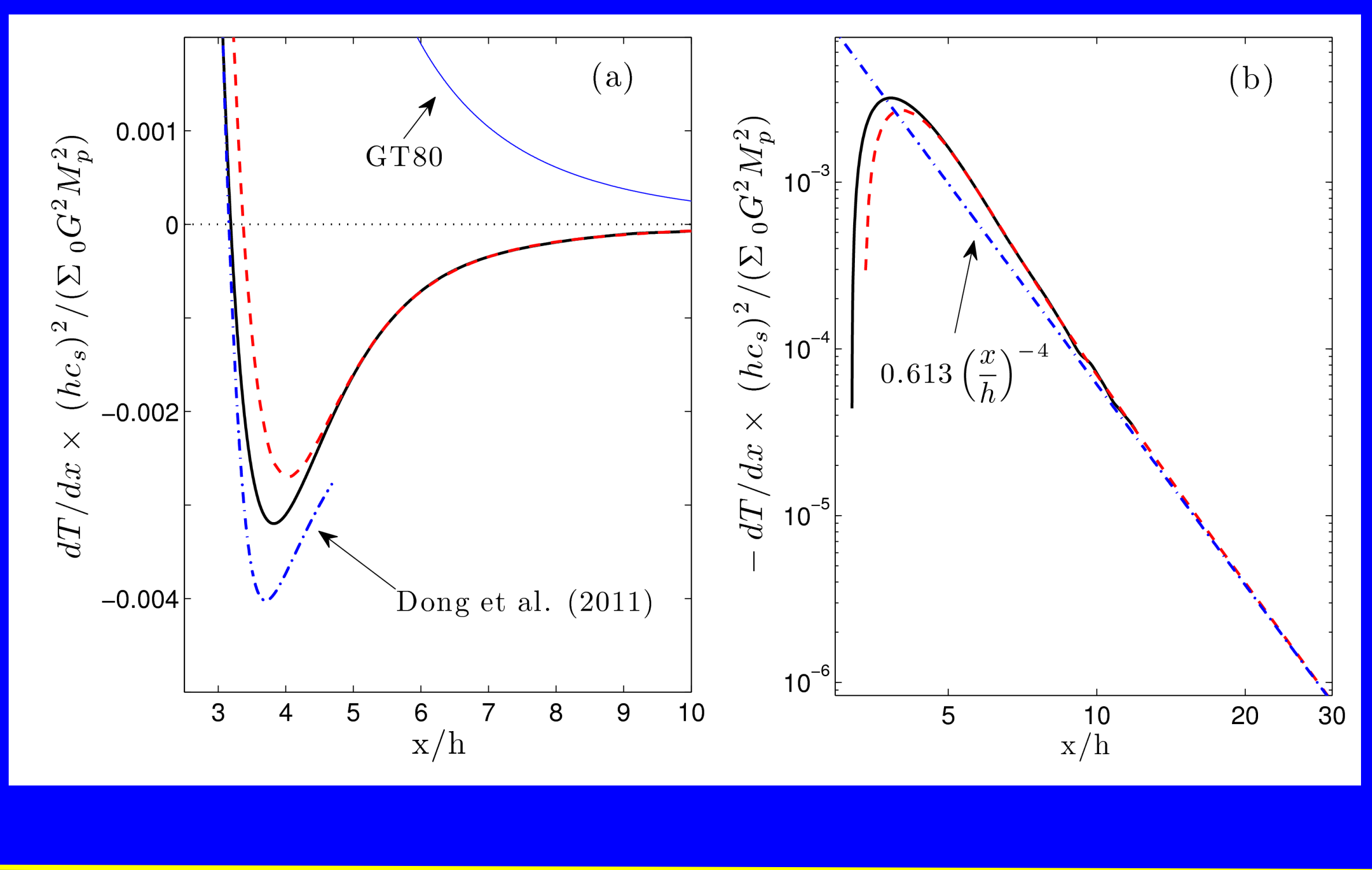
New (our work) asymptotic torque density

$$\left(\frac{dT}{dx}\right)^{as}\to{\rm sgn}(x)C\frac{(GM_p)^2\Sigma_0}{\Omega^2}\frac{1}{x^4},$$
$$C=\frac{16}{81}\left[2K_1\left(\frac{2}{3}\right)-5K_0\left(\frac{2}{3}\right)\right]\left[2K_0\left(\frac{2}{3}\right)+K_1\left(\frac{2}{3}\right)\right]\approx-0.613096.\qquad (2$$



Rafikov & Petrovich (2012) showed that this change in sign of the excitation torque density occurs because of the overlap of Lindblad resonances – their width is larger than the separation between them, which causes non-trivial interference of different resonances. Previous studies did not recognize this effect as the strength of resonances computed in Fourier space was projected onto the physical space without accounting for the resonance overlap effect. The effect is rather small – only several % of the full torque exerted by the planet is accumulated in the region of the negative torque density. More important is its qualitative novelty.

Torque density in disks with gaps

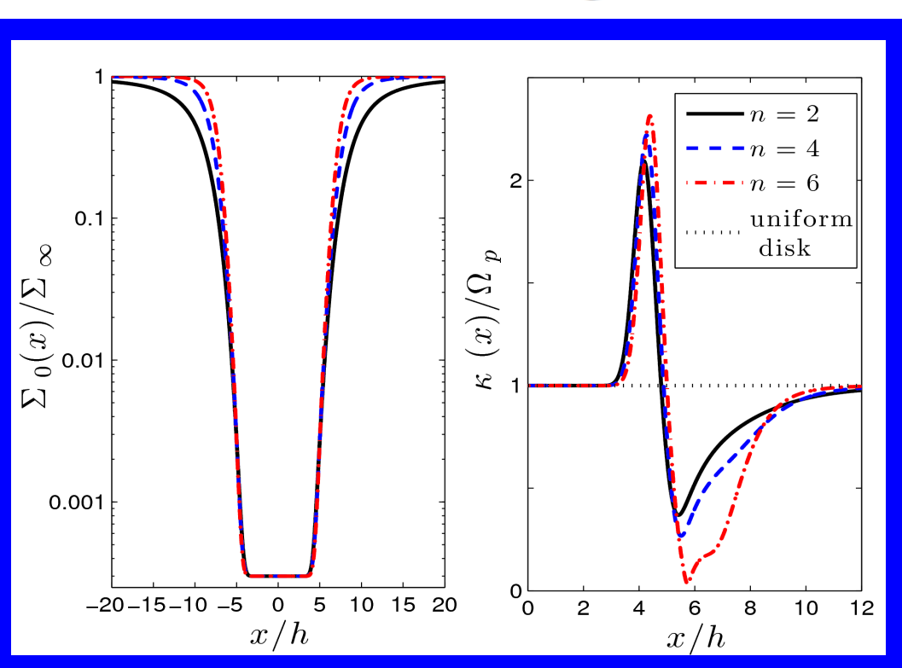
Petrovich & Rafikov (2012) explored the behavior of torque density in a disk with a gap around the planetary orbit. According to conventional wisdom, torque density in a non-uniform disk is computed via convolution with the torque density in the uniform disk, assumed to be given by the GT80 prescription.

$$\frac{dT(r)}{dr}=\frac{\Sigma(r)}{\Sigma_0}\left.\frac{dT(r)}{dr}\right|_u$$

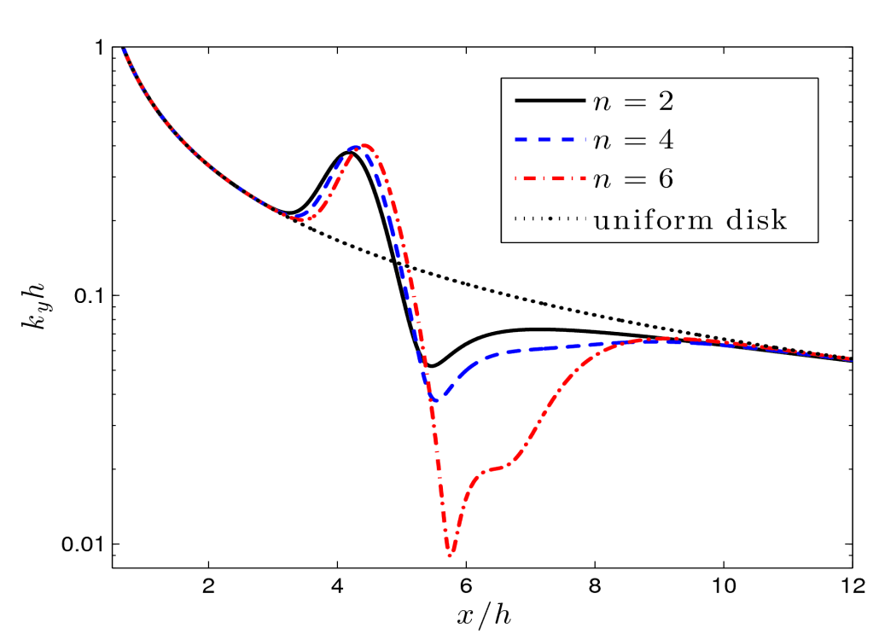
Negative torque density phenomenon makes this assumption very problematic for broad gaps.
We addressed this problem via the linear calculation fully incorporating disk inhomogeneity in fluid equations. This introduces new terms in the equations for fluid perturbations (absent in the case of homogeneous disk) and results in modified eigenfunctions for different azimuthal harmonics of the planetary potential. Because of the pressure support arising in a non-uniform disk, epicyclic frequency becomes different from the Keplerian frequency. This displaces the locations of the Lindblad resonances compared to the standard case and changes their number and strength. Coupled with proper account of the resonance interference all this results in a different behavior of the torque density in the non-uniform disks (see figures below). In particular, we found that outside the gap torque density decays with distance exponentially, rather than as a power law (as is usually assumed). These results are very important for self-consistent calculation of the gap shapes.

Density and kappa runs

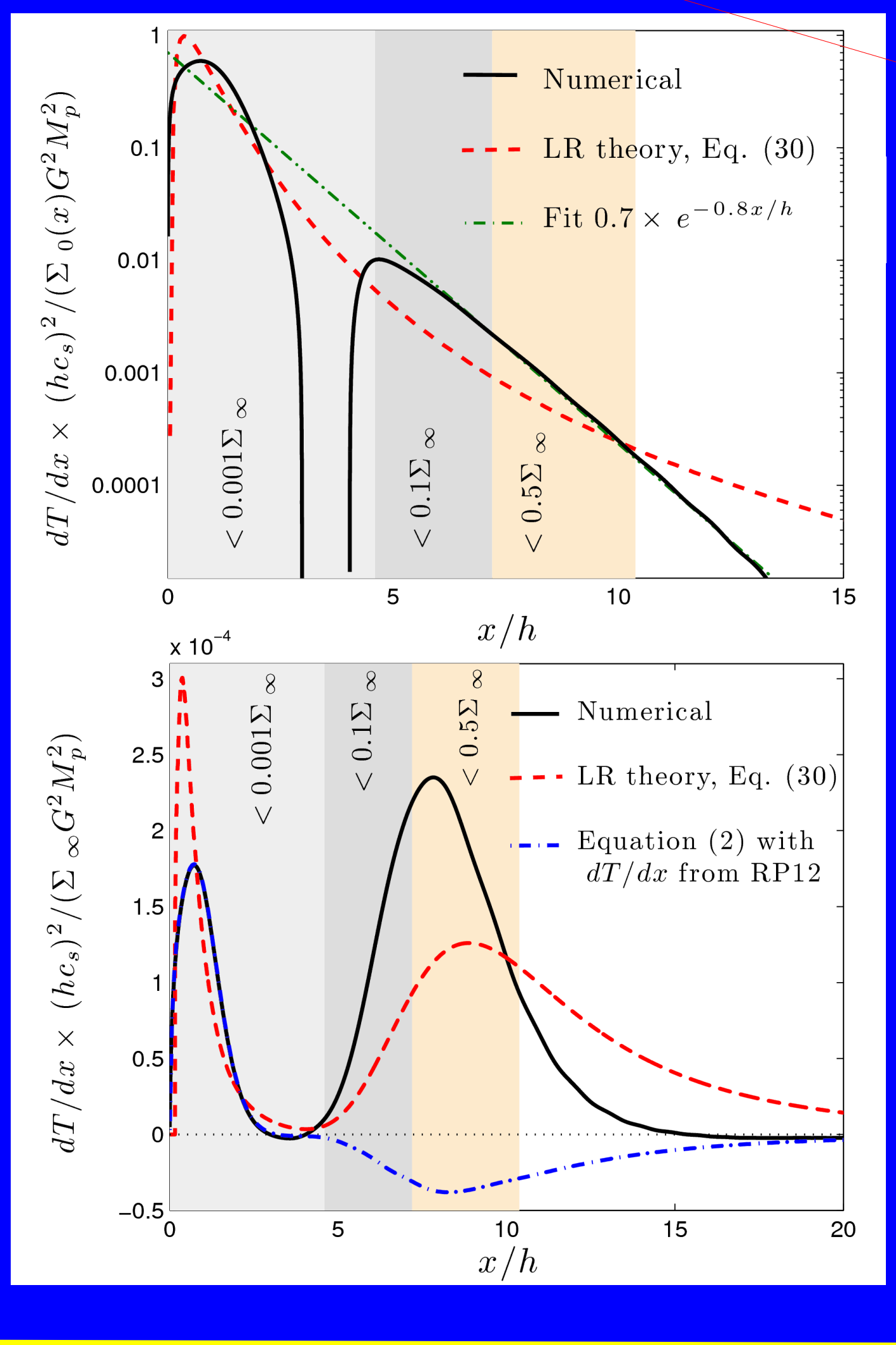
$$\frac{\Sigma_0(x)}{\Sigma_\infty}=1-\frac{1-\Sigma_{\rm min}}{1+(x/\delta)^n\exp[-(2.5\delta/x)^2]}$$



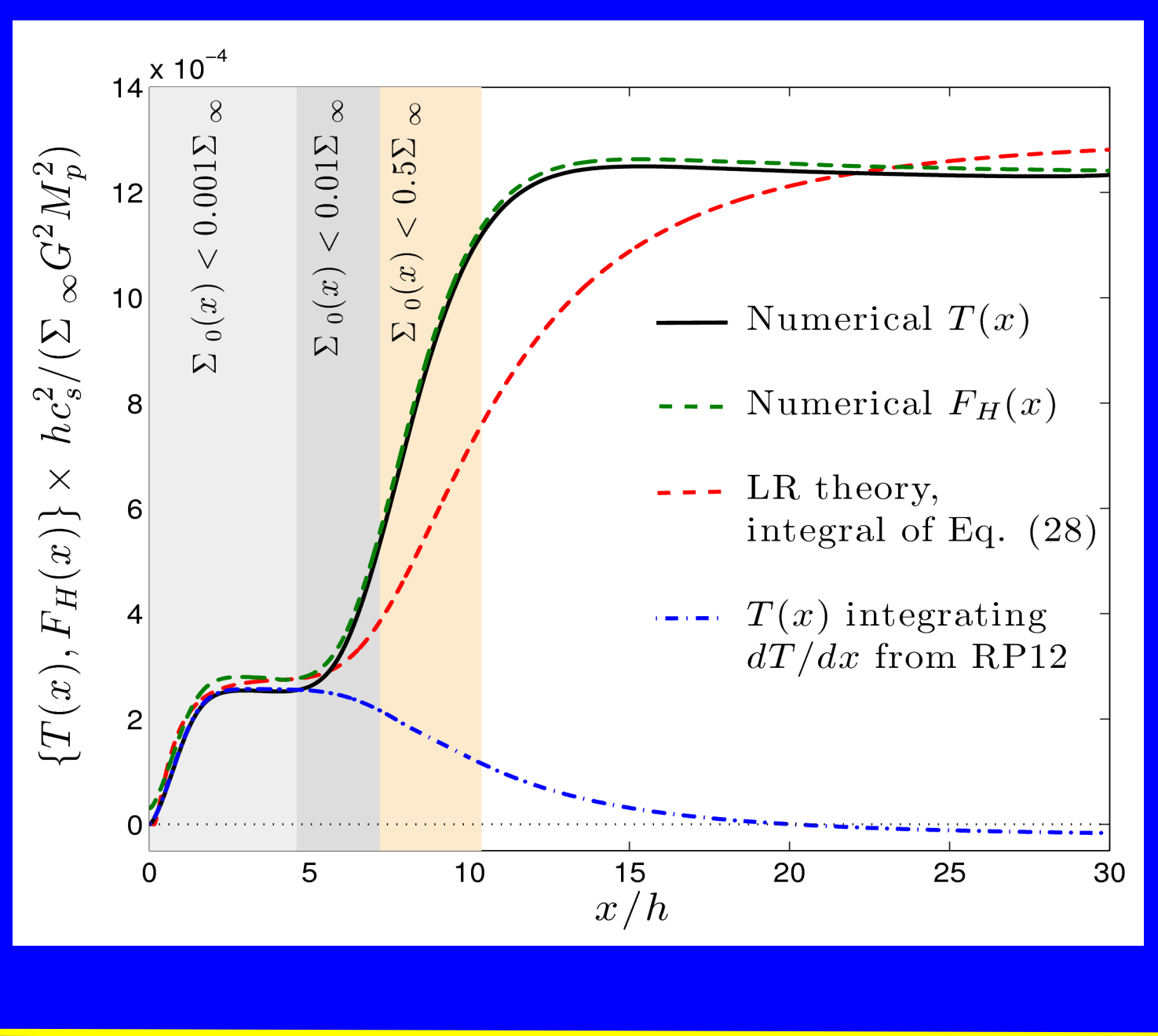
Lindblad resonance positions



Torque density behavior in a disk with the gap



$$\frac{dT}{dx}=C(\alpha){\rm sign}(x)\frac{(GM_p)^2\Sigma_0(x)}{\Omega^2h^4}e^{-\alpha x/h},$$
$$C(\alpha)=2.5\frac{\int_\delta^\infty\Sigma_0(x)/(x/h)^4dx}{\int_\delta^\infty\Sigma_0(x)e^{-\alpha x/h}dx},$$



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