

Critical core mass and the abundance of heavy elements

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Introduction

Within the core accretion scenario of planetary formation, simulations performed so far assume that the planetesimals are accreted by protoplanets without undergoing any physical disruption during their way to the core. This implies that during the growth of a giant planet, all the solids are deposited in the core and do not modify the primordial H-He envelope. In this work, we begin to investigate the physical implications of taking into account the change of composition of the envelope of proto-giant planets due to the ablation of planetesimals. A series of static internal structure models are run with the implementation of the free software CEA to compute the equation of state for the polluted envelope, and updated opacities for all ranges of metallicities. We show that a consequence of incorporating a non-solar metallicity in the envelope of protoplanets is a strong reduction in the critical core mass.

Code features:

Solves the static internal structure equations assuming spherical symmetry and a constant accretion rate of solids (or luminosity). For better numerical performance, the equations are written as a function of $v = r^3/M_r$:

$$\frac{dP}{dv} = -\rho \left(\frac{GM_r}{v^2} \right)^{2/3} [3 - 4\pi\rho v]^{-1} \quad (\text{Hydrostatic equilibrium})$$

$$\frac{dM_r}{dv} = 4\pi\rho M_r [3 - 4\pi\rho v]^{-1} \quad (\text{Mass conservation})$$

$$\frac{dT}{dv} = \frac{T}{P} \frac{dP}{dv} \nabla \quad (\text{Heat transport})$$

Where $\nabla \equiv \frac{\partial \ln T}{\partial \ln P}$ \rightarrow $\left\{ \begin{array}{l} \nabla_{\text{rad}} = \frac{3\kappa LP}{64\pi\sigma GM_r T^4} \\ \nabla_{\text{ad}} = \left(\frac{\partial \ln T}{\partial \ln P} \right)_s \end{array} \right.$

(Convection occurs if $\nabla_{\text{ad}} < \nabla_{\text{rad}}$)

The luminosity is considered uniform throughout the envelope, and is given as:

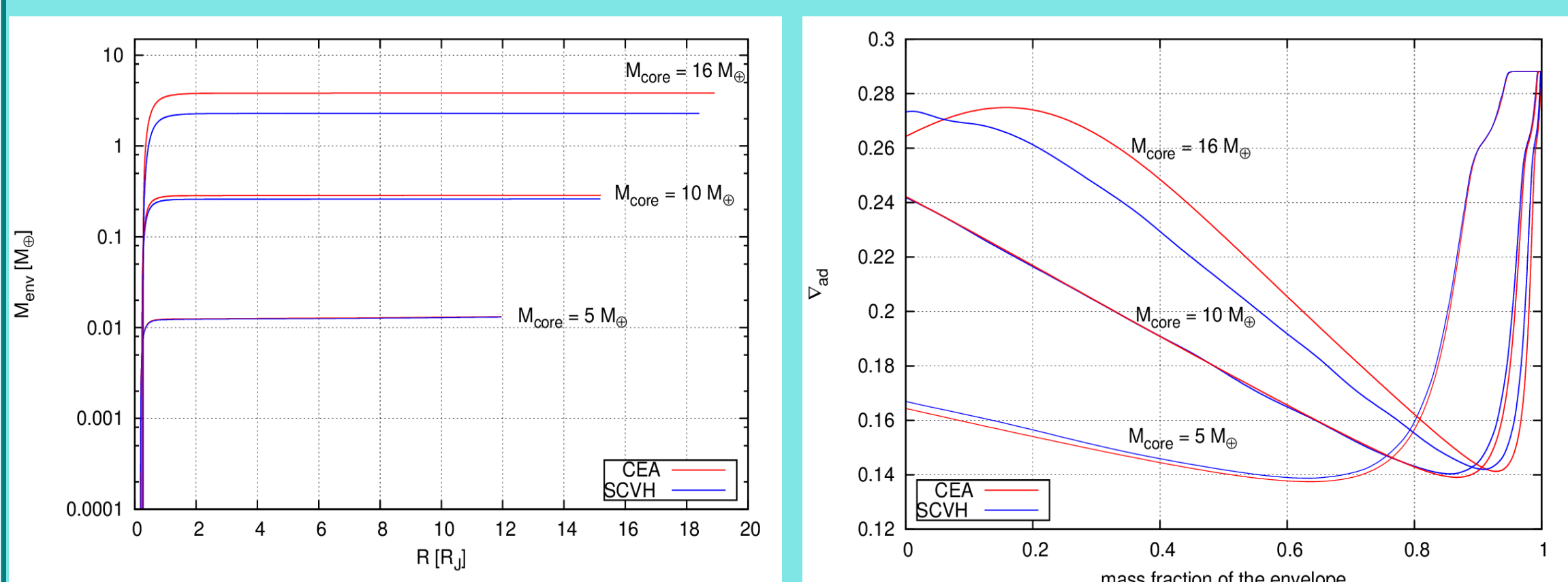
$$L = \frac{GM_c \dot{M}_c}{R_c}$$

- **Opacities:** Bell & Lin (1994), Ferguson (1994) (for gas and all ranges of metallicities), Semenov (2003) (for dust)
- **EOS:** Saumon et al. (1995) (for H-He), CEA (for any arbitrary composition)

About CEA (Chemical Equilibrium Applications):

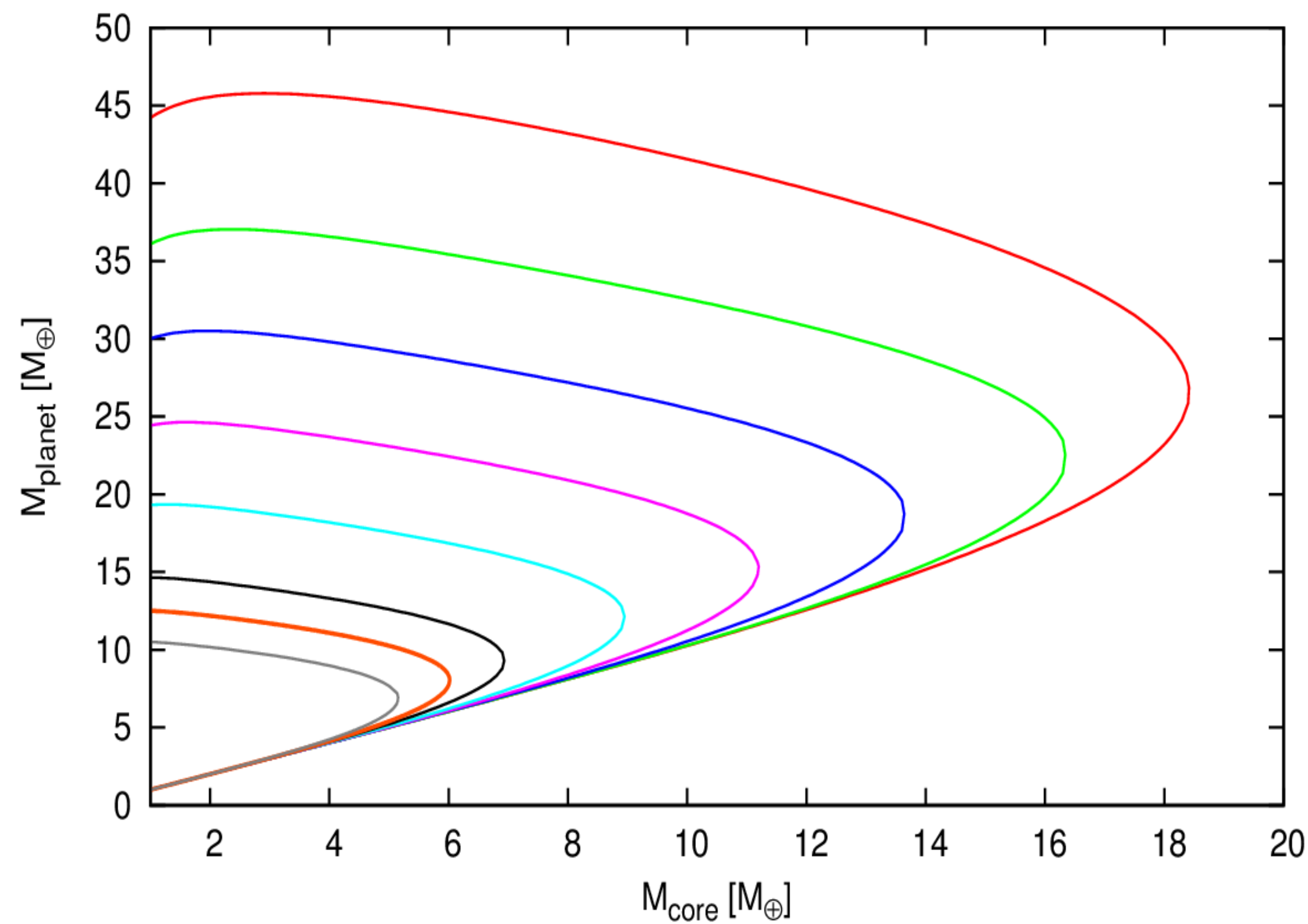
Solves the chemical equilibrium equations for an arbitrary defined mixture. It assumes an ideal equation of state for the mixture, but taking into account the proper dissociation and ionization temperatures of each compound formed.

Profiles for planets with different core mass and H-He envelope using CEA and Saumon et al. (1995) (SCVH) EOS:



References:

- Alexander, D. R., & Ferguson, J. W. 1994, ApJ, 437, 879
- Bell, K. R., & Lin, D. N. C. 1994, ApJ, 427, 987
- Gordon & McBride, 1994, NASA Reference Publication 1311.
- Hori, Y & Ikoma, M. 2011, MNRAS, 416, 1419.
- Papaloizou & Terquem, 1999, ApJ, 521, 823.
- Saumon, D., Chabrier, G., & Van Horn, H. M. 1995, ApJS, 99, 713.
- Semenov, D. et al. 2003, A&A, 410, 611.



Z=0, SCVH (red), Z=0.1 (blue), Z=0.3 (cyan), Z=0.45 (orange), Z=0.5 (grey)
Z=0, CEA (green), Z=0.2 (magenta), Z=0.4 (black)

Fig. 1: For a given core mass, uniform metallicity (Z) and accretion rate of solids of $10^{-6} M_{\text{Earth}}/\text{yr}$, the total mass of the planet is computed solving the internal structure equations, with BL opacities. Boundary conditions:
• $a = 0.5 \text{ AU}$
• $T_{\text{out}} = 1063.53 \text{ K}$
• $P_{\text{out}} = 42.19 \text{ dyn/cm}^2$ (as in case 4 of Papaloizou and Terquem, 1999).

For the EOS, the CEA program was used. In spite of not taking into account the degeneracy of the gas (compare curve red with green for zero metallicity and also previous plots to have an idea of the effect of degeneracy), a clear trend of reduction in the critical core mass is observed when Z increases in the envelope.

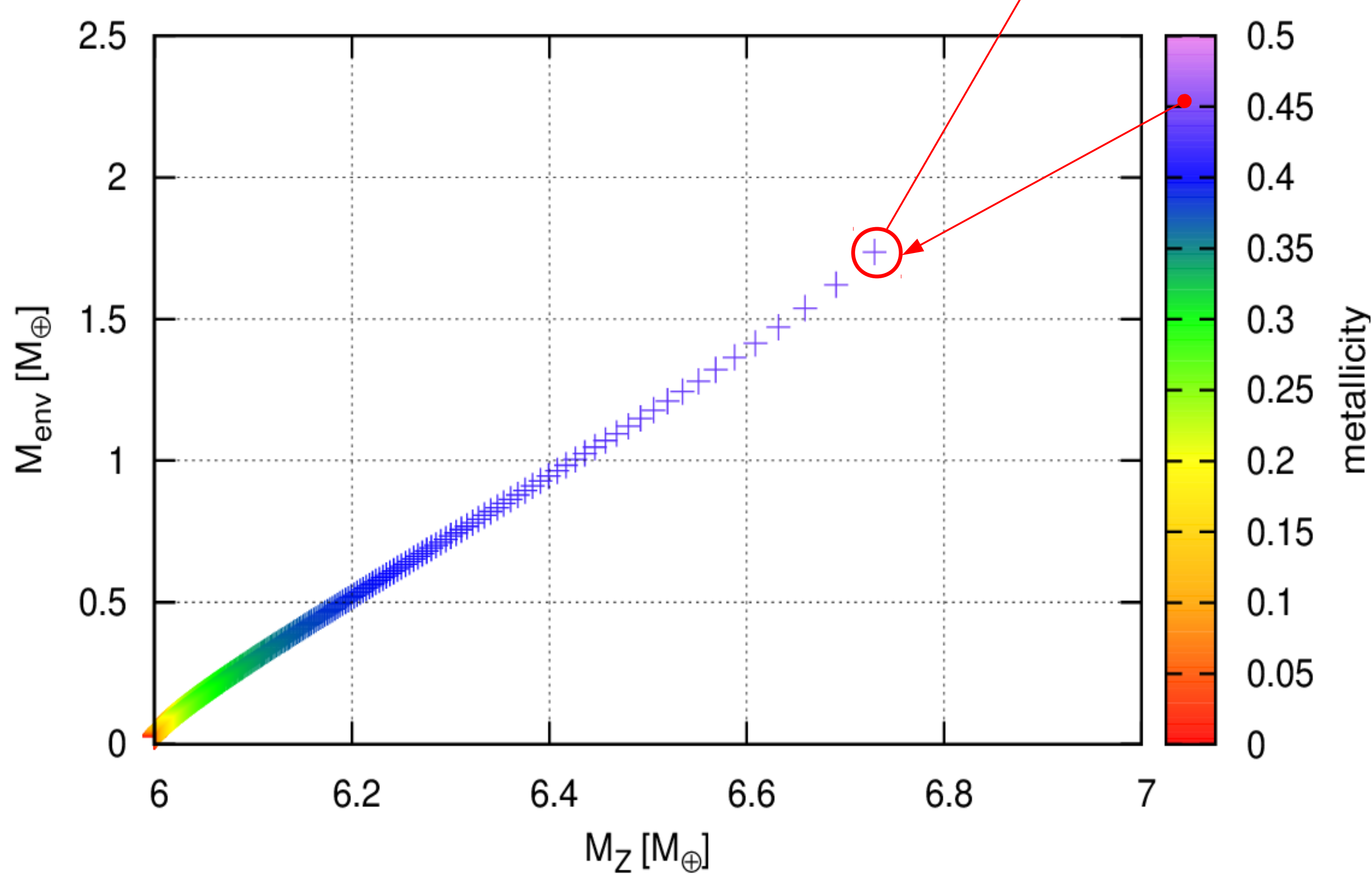


Fig. 2: Same boundary conditions as before, but in this case the solids are sent to the core up to a mass of $6 M_{\text{Earth}}$, and after this they are considered to remain in the envelope, increasing its Z . (M_z is the total mass of solids, that is, $M_z = M_{\text{core}} + Z M_{\text{env}}$). The last point of the plot (inside the red circle) corresponds to the critical value of a core mass of $6 M_{\text{Earth}}$ and an envelope metallicity of 0.45 (see Fig. 1)

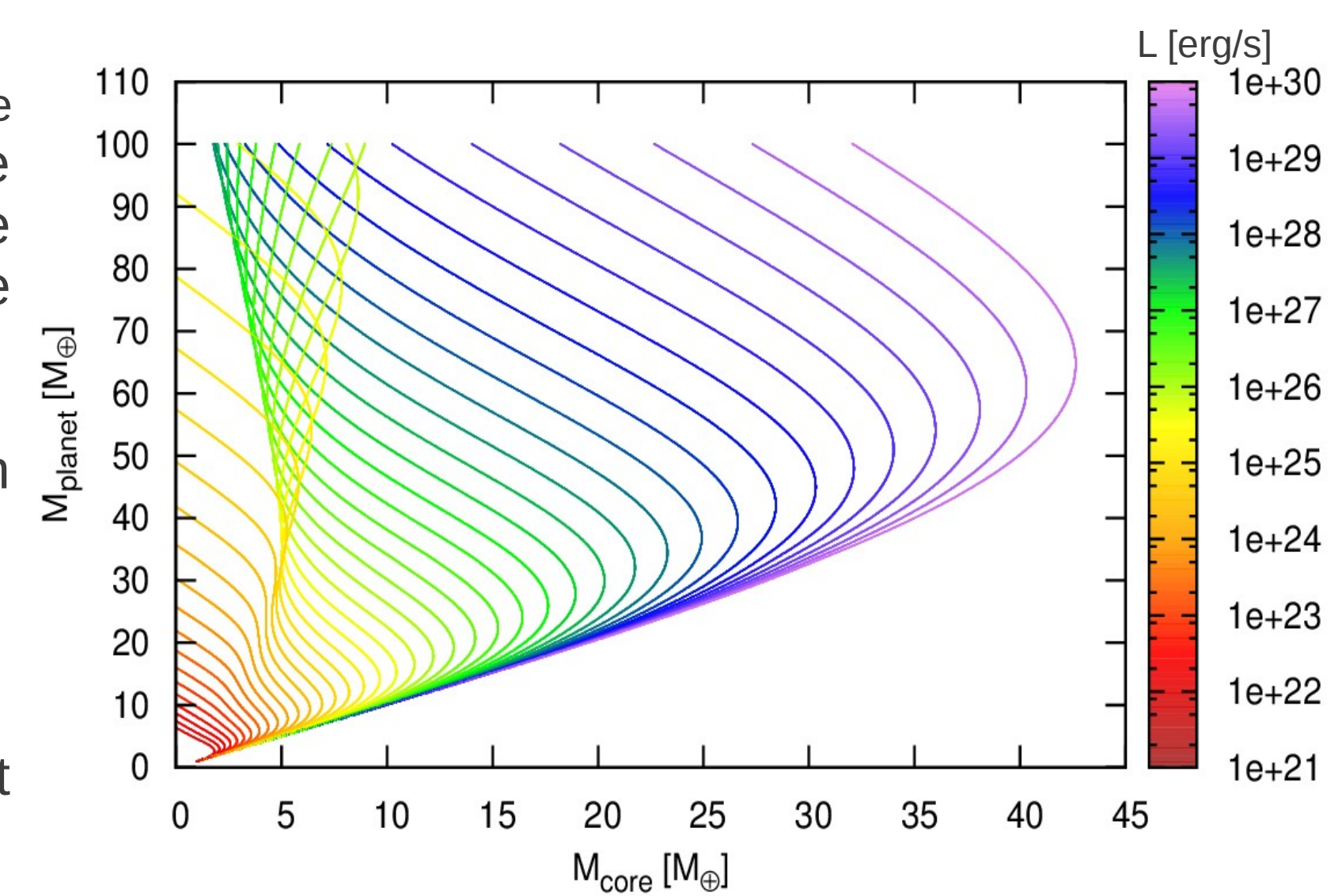
Fig. 3: Curves of constant luminosity in a $M_{\text{planet}}-M_{\text{core}}$ diagram. In a realistic growth scenario, the planet will move through a path in this plot, jumping from one L to a larger one as it accretes more planetesimals and as the envelope contracts.

If a constant accretion rate of solids is assumed, the evolution can be computed considering:

$$dt = \frac{\Delta M_c}{\dot{M}_c} \quad L_{\text{gas}} = -\frac{dE_{\text{env}}}{dt}$$

$$L_{\text{core}} = \frac{GM_c \dot{M}_c}{R_c} \quad L_{\text{tot}} = L_{\text{core}} + L_{\text{gas}}$$

Then, for a given L , the planet will be situated in the point where $L_{\text{tot}} = L$



Conclusions:

- The critical core mass is dramatically reduced when the effect of change of composition of the envelope due to ablated planetesimals is taken into account.
- This effect is more important concerning the change in mean molecular weight and adiabatic gradient (ie., the change in the EOS when a Z -component is included) than the change in opacity (with respect to solar values).

Work in progress:

- Computation of S_{mix} , using CEA for the Z -component of the envelope and SCVH for the H-He component.
- Evolution code: growth of the planet with a polluted envelope. The planet will follow a path in a diagram like the one in Fig. 3.