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On the Minimum Core Mass for Giant Planet Formation

ABSTRACT

The core accretion model proposes that giant planets form by the accretion of gas onto a solid protoplanetary core. Previous studies have found that there exists a "critical core mass" past which hydrostatic solutions can no longer be found and unstable atmosphere collapse occurs. This core mass is typically quoted to be around $10M_\oplus$. In standard calculations of the critical core mass, planetesimal accretion deposits enough heat to alter the luminosity of the atmosphere, increasing the core mass required for the atmosphere to collapse. In this study we consider the limiting case in which planetesimal accretion is negligible and Kelvin-Helmholtz contraction dominates the luminosity evolution of the planet. We develop a two-layer atmosphere model with an inner convective region and an outer radiative zone that matches onto the protoplanetary disk, and we determine the minimum core mass for a giant planet to form within the typical disk lifetime for a variety of disk conditions. We denote this mass as critical core mass. The absolute minimum core mass required to nucleate atmosphere collapse is ~ $8M_e$ at 5 AU and steadily decreases to ~ $3.5M_e$ at 100 AU, for an ideal diatomic gas with a solar composition and a standard ISM opacity law. Lower opacity and disk temperature significantly reduce the critical core mass, while a decrease in the mean molecular weight of the nebular gas results in a larger critical core mass. Our results yield lower mass cores than corresponding studies for large planetesimal accretion rates.

TWO-LAYER ATMOSPHERE MODEL

- Two-layer atmosphere model, consisting of an inner convective region and an outer radiative region that matches onto the disk.
- Assumes that the protoplanetary core no longer accretes planetesimals and remains at constant mass.
- The atmosphere is considered to be in hydrostatic equilibrium.
- Under these conditions, the structure of the atmosphere is determined by the following equations:

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho \qquad \frac{dT}{dr} = \nabla\frac{T}{P}\frac{dP}{dr}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho \qquad \frac{dL}{dr} = 4\pi r^2 \rho \varepsilon_g$$

$$\nabla = \begin{cases} \nabla_{ad} = \left(\frac{d\ln T}{d\ln P}\right)_{ad}, & \nabla_{ad} < \nabla_{rad} \\ \nabla_{rad} = \frac{3\kappa P}{64\pi G m \sigma T^4} L, & \nabla_{ad} \geq \nabla_{rad} \end{cases}$$

- The luminosity is assumed to be constant throughout the radiative region of the atmosphere: $\varepsilon_g = -T\, dS/dt = 0$
- Boundary conditions: we assume the atmosphere extends out to the boundary of the Hill sphere, where it matches smoothly on to the disk: $T(R_H)=T_d$, $P(R_H)=P_d$.
- Equation of state: first ideal gas polytrope with μ = 2.35 and ∇_{ad} = 2/7; then realistic EOS tables (Saumon et al. 2005)
- Opacity: dust power law opacity $\kappa=\kappa_0 T^\beta P^\alpha$, α=0, β~2

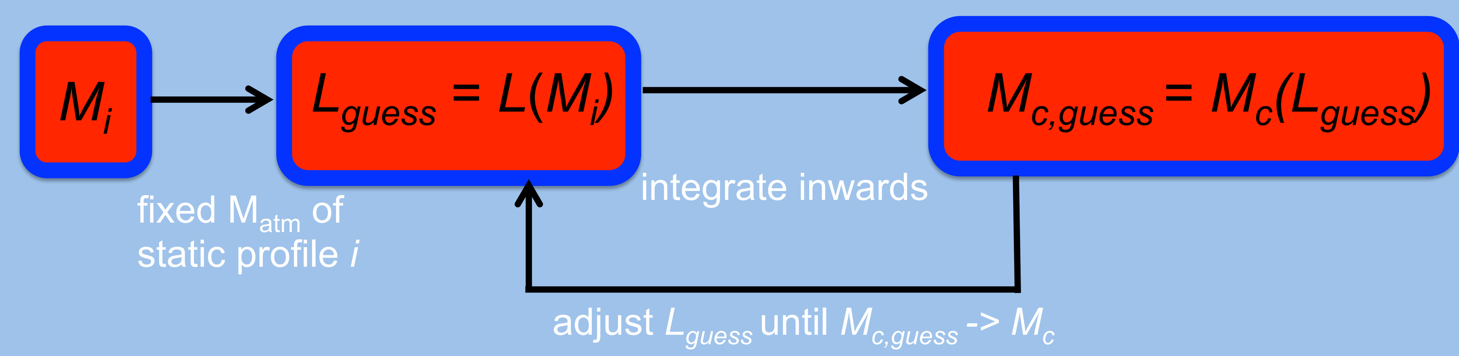
ENERGETICS AND COOLING MODEL

- In order to determine the time evolution of the atmosphere, we use a global cooling model for a protoplanetary atmosphere embedded in a gas disk:

$$L = L_c + \Gamma - \dot{E} + e_{acc}\dot{M} - P_M \frac{\partial V_M}{\partial t}$$

- The cooling model applies at any radius *R* where the mass enclosed is *M*. This radius can be the Hill radius, the Bondi radius, or at the boundary between the radiative and convective regions of the atmosphere (RCB).

NUMERICAL METHOD



$$\Delta t_{i,i+1} = \frac{-\Delta E_i + e_{acc,i}\Delta M_i - P_i \Delta V_i}{L_i}$$

RESULTS

ATMOSPHERE PROFILES

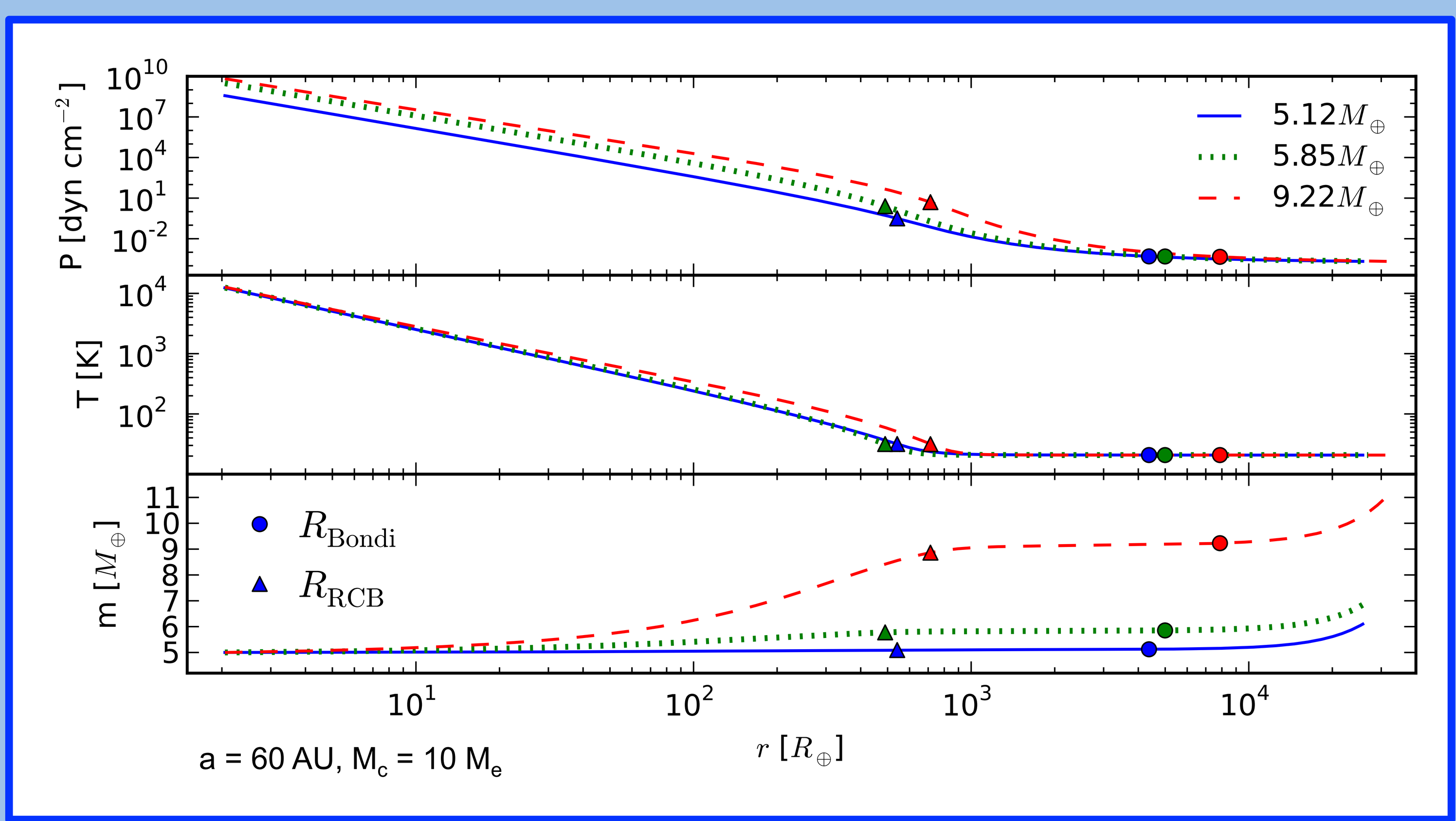


Fig. 1: Example pressure, temperature and mass profiles as a function of radius.

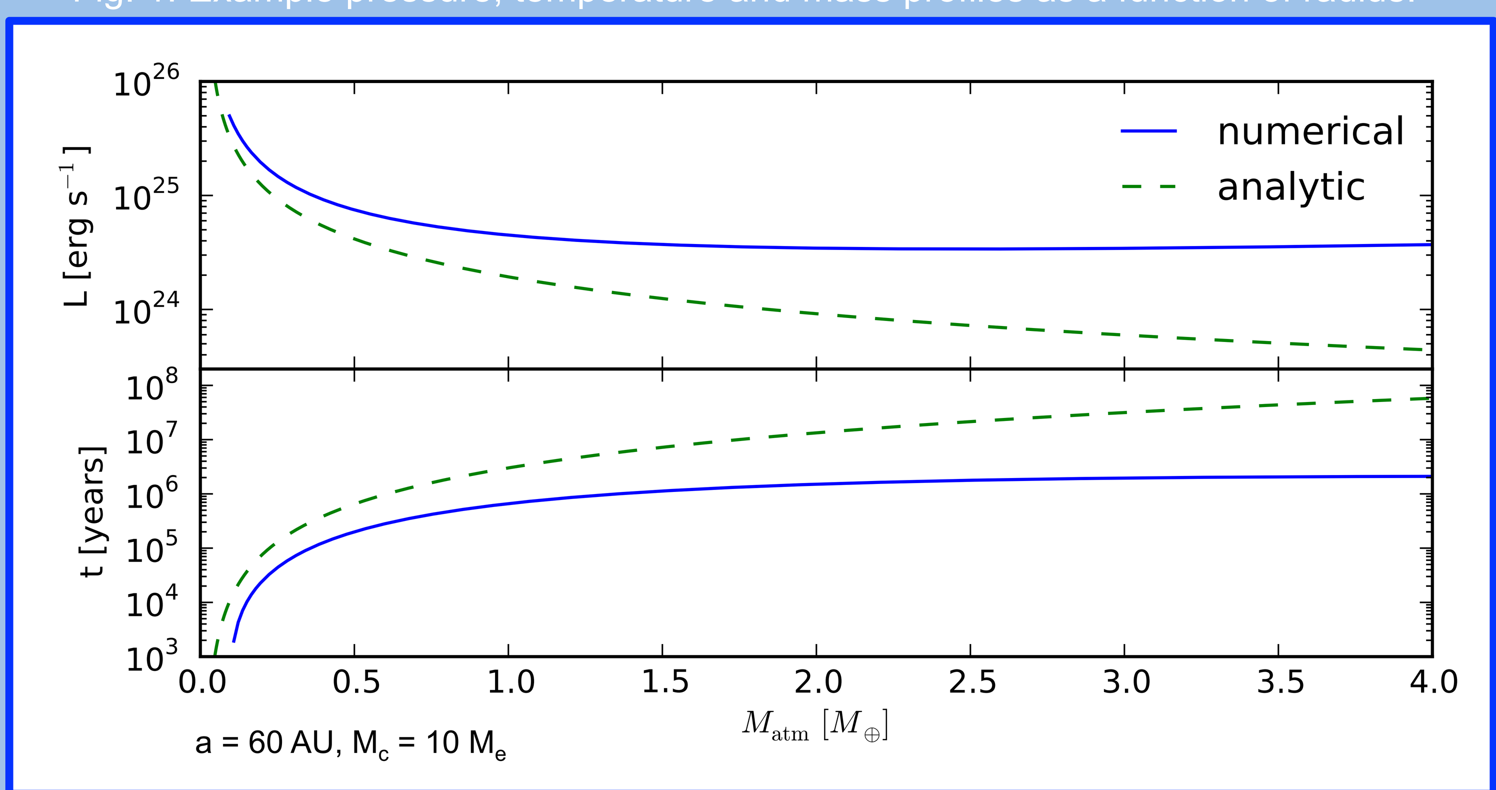


Fig. 2: Luminosity and time evolution with mass. Both Fig. 1 and Fig. 2 are for an ideal diatomic gas and an ISM opacity.

CRITICAL CORE MASS

- As the atmosphere mass becomes roughly the same as the core mass, runaway accretion commences. We define the time at which $M_{atm} \sim M_c$ as the *crossover time*.

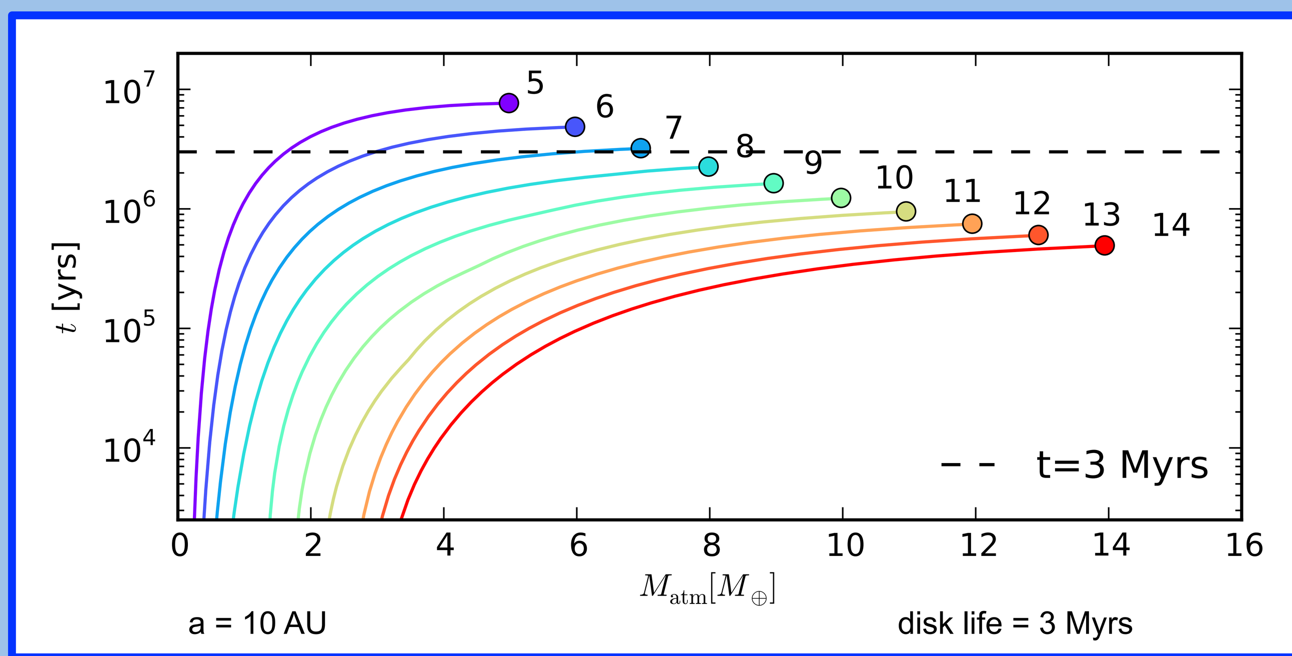


Fig. 3: Time evolution as a function of atmosphere mass for a range of core masses, for an ideal diatomic gas and for an ISM opacity law. The circles mark the crossover time.

- Once the time evolution is obtained, we are interested in knowing the minimum core mass for an atmosphere to form within the life time of the protoplanetary disk (typically of the order of few million years). We define this mass as the *critical core mass*.

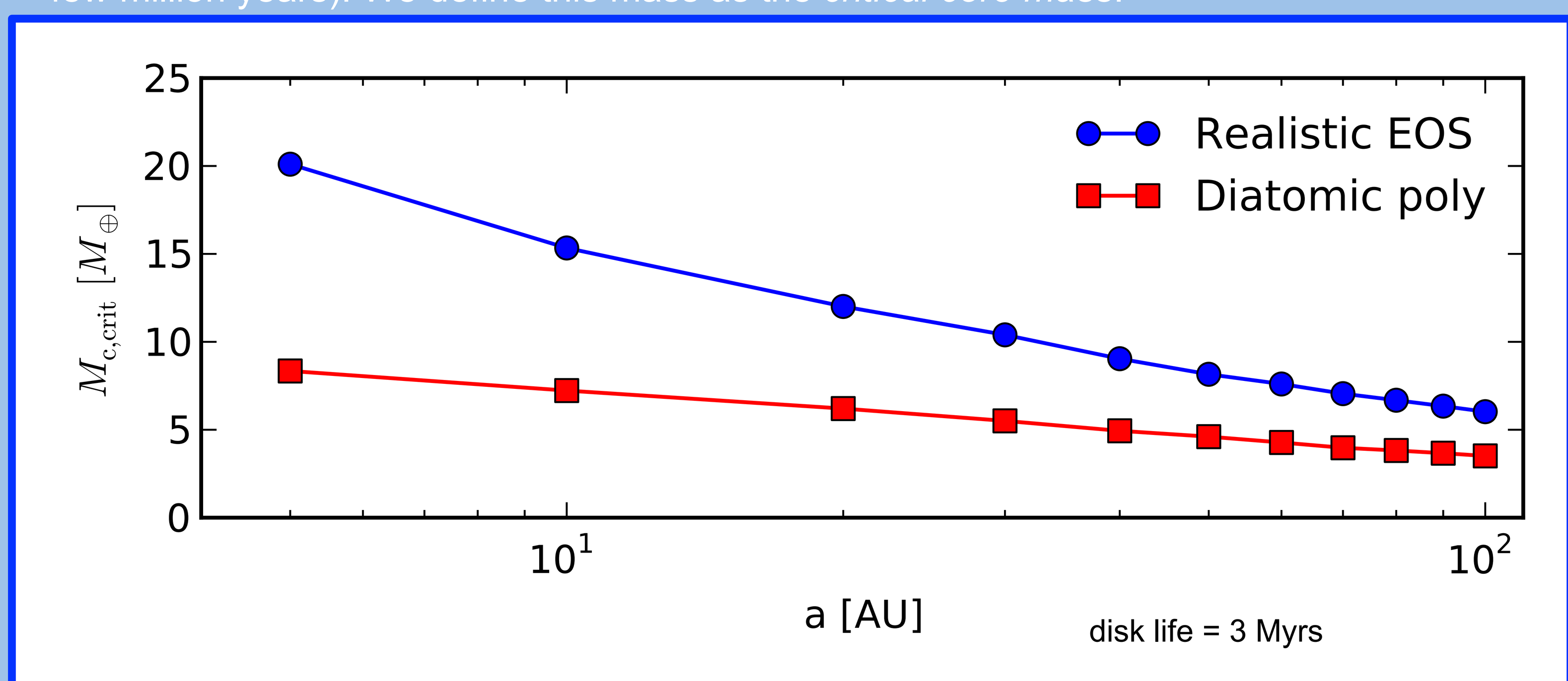


Fig. 4: The minimum core mass for an atmosphere to become 'critical' within the lifetime of the disk as a function of semi-major axis, for and ideal diatomic gas and a gas described by a realistic equation of state. For a lower, more realistic opacity (e.g., due to grain growth), see Figure 5.

COMPARISON WITH OTHER STUDIES

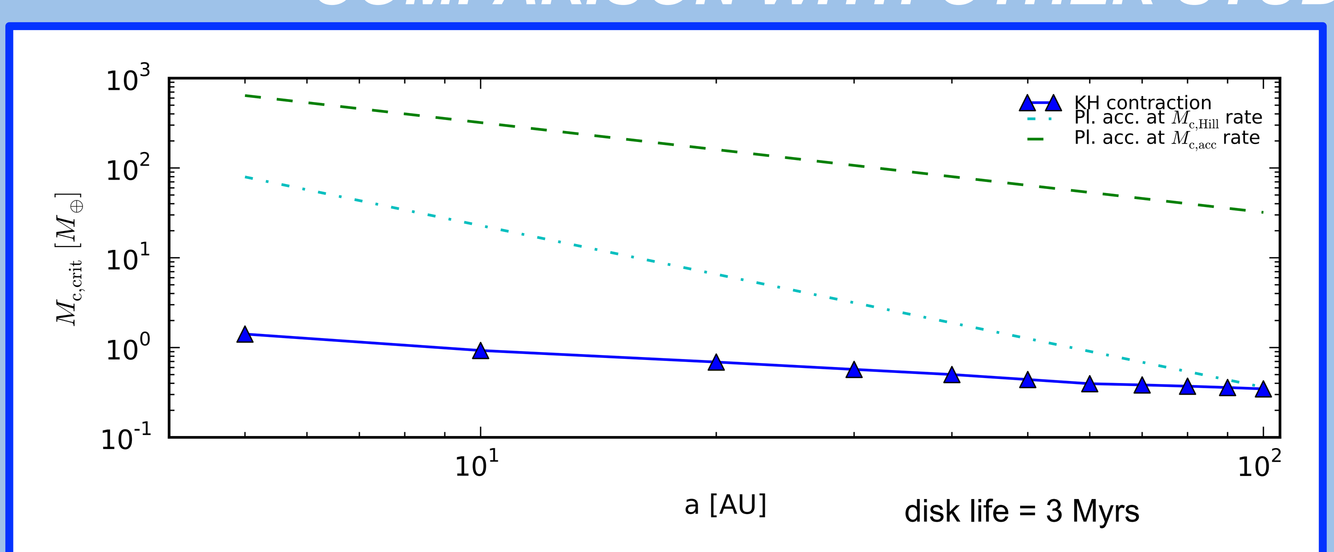


Fig. 5: Comparison between the critical core mass Mcrit,acc due to planetesimal accretion and the assumed fixed core mass when gas contraction dominates, for a growth time of T = 2.6 Myrs. The dashed line is based on Rafikov (2006), for opacity and disk assumptions similar to ours. Our results yield lower core masses than in the standard case. Additionally, a lower opacity lowers M_{crit}.

CONCLUSIONS

- We find that the critical core mass to form a giant planet before the dissipation of the protoplanetary disk is smaller than typical values of the critical core mass when planetesimal accretion dominates the atmosphere growth (e.g., Rafikov 2006). Growing the core first, then reducing the core accretion rate, results in a more effective atmospheric growth. Moreover, our results represent a true minimum for the core mass needed to form a giant planet during the disk lifetime.
- The non-ideal effects of a realistic gas increase the critical core mass by more than a factor of two.
- The minimum core mass necessary to form a giant planet is smaller for planets forming further out in the protoplanetary disk.

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