



Abstract

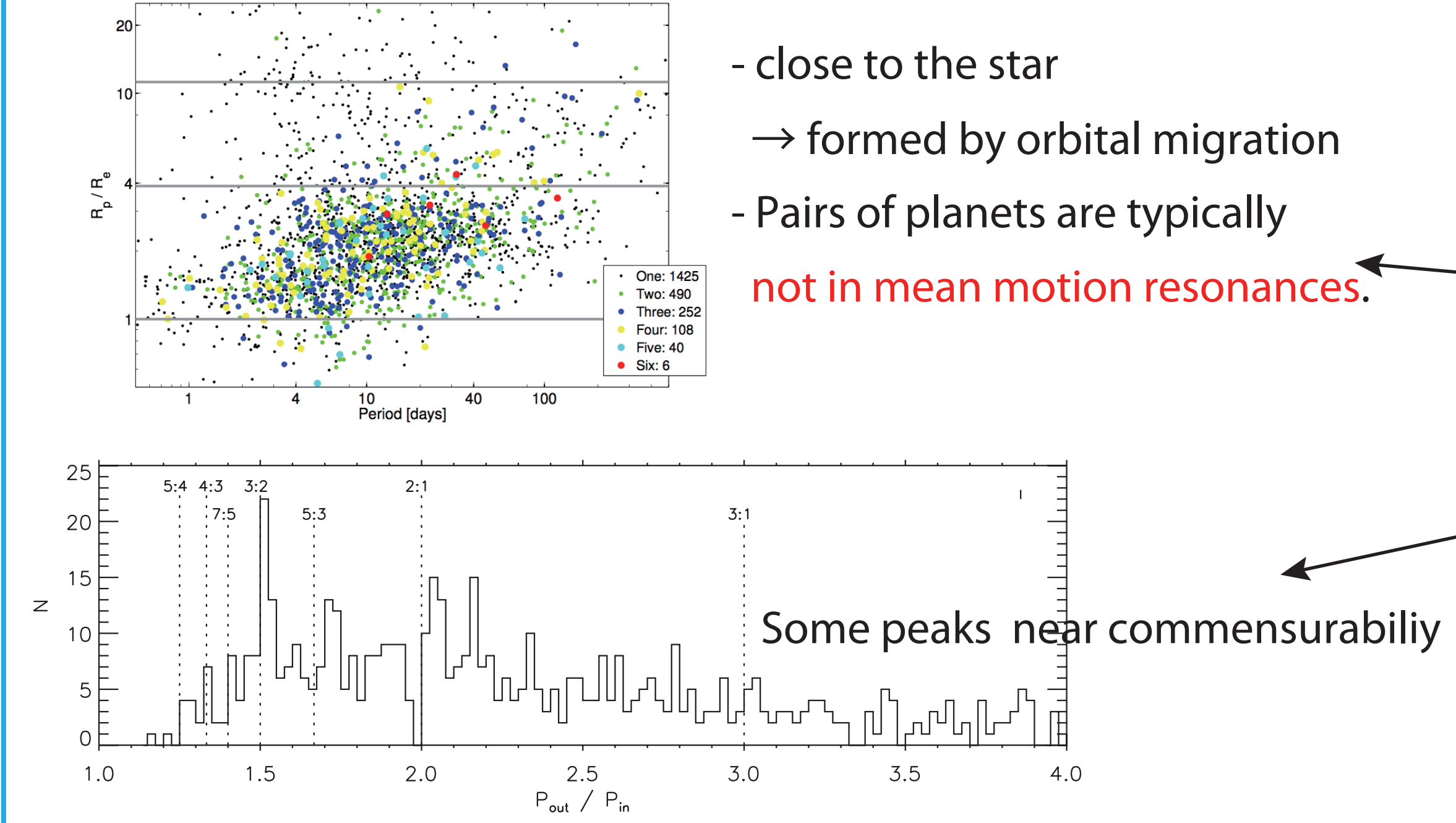
Kepler Mission suggests that period ratios of pairs of planets show some peaks at commensurable ratios (e.g., Fabrycky et al., 2012). Some N-body simulations considering planetary migration suggest protoplanets are captured in mean-motion resonant orbits. These resonant systems are stable in the calculations. However in some cases, resonant systems cause orbital instability and finally non-resonant systems are formed.

So, we investigated the orbital stability of first-order resonant systems to clarify what makes resonant systems unstable. We found in resonant systems, when the number of planets is more than the critical number, the crossing time decreases rapidly, while in non-resonant system, the crossing time decreases continuously. We also investigated the parameter dependence of the critical number.

Introduction

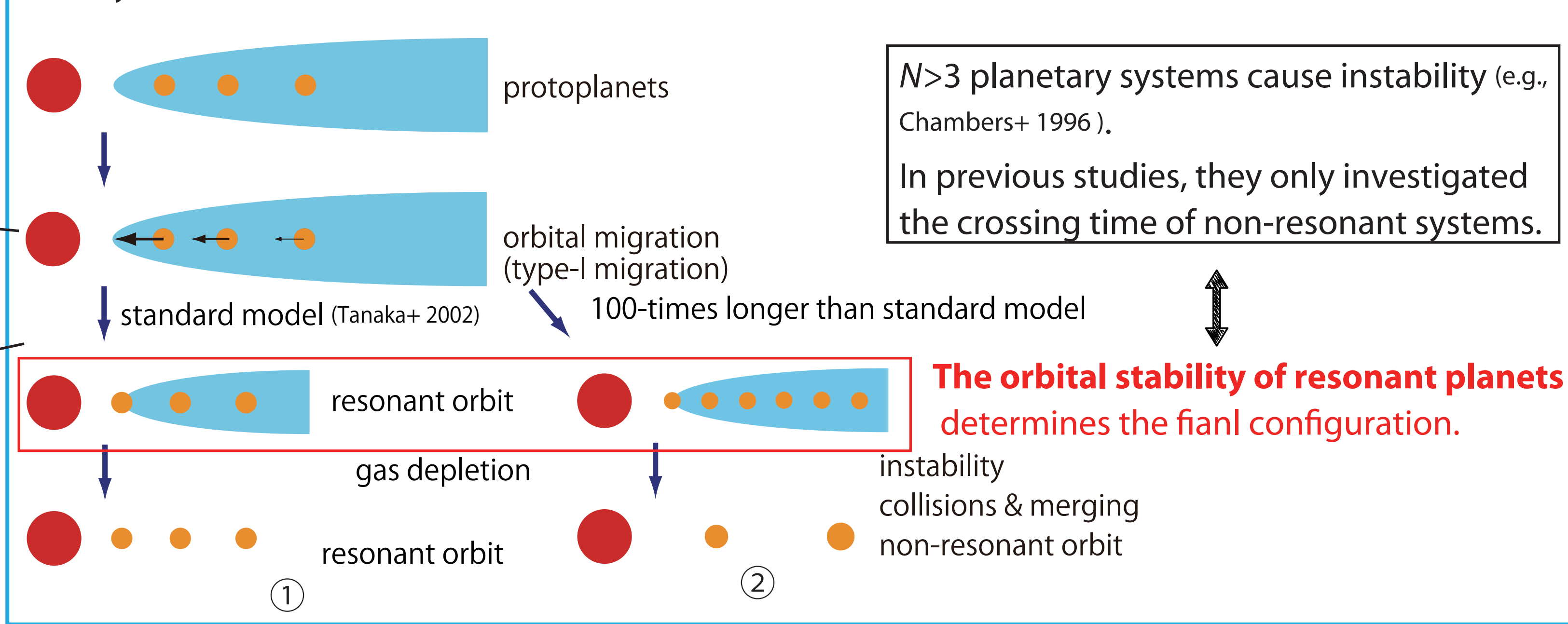
Observed Planetary Systems

KOI candidates (Batalha+ 2011, Fabrycky+ 2012)



N-body Simulations with Orbital Migration

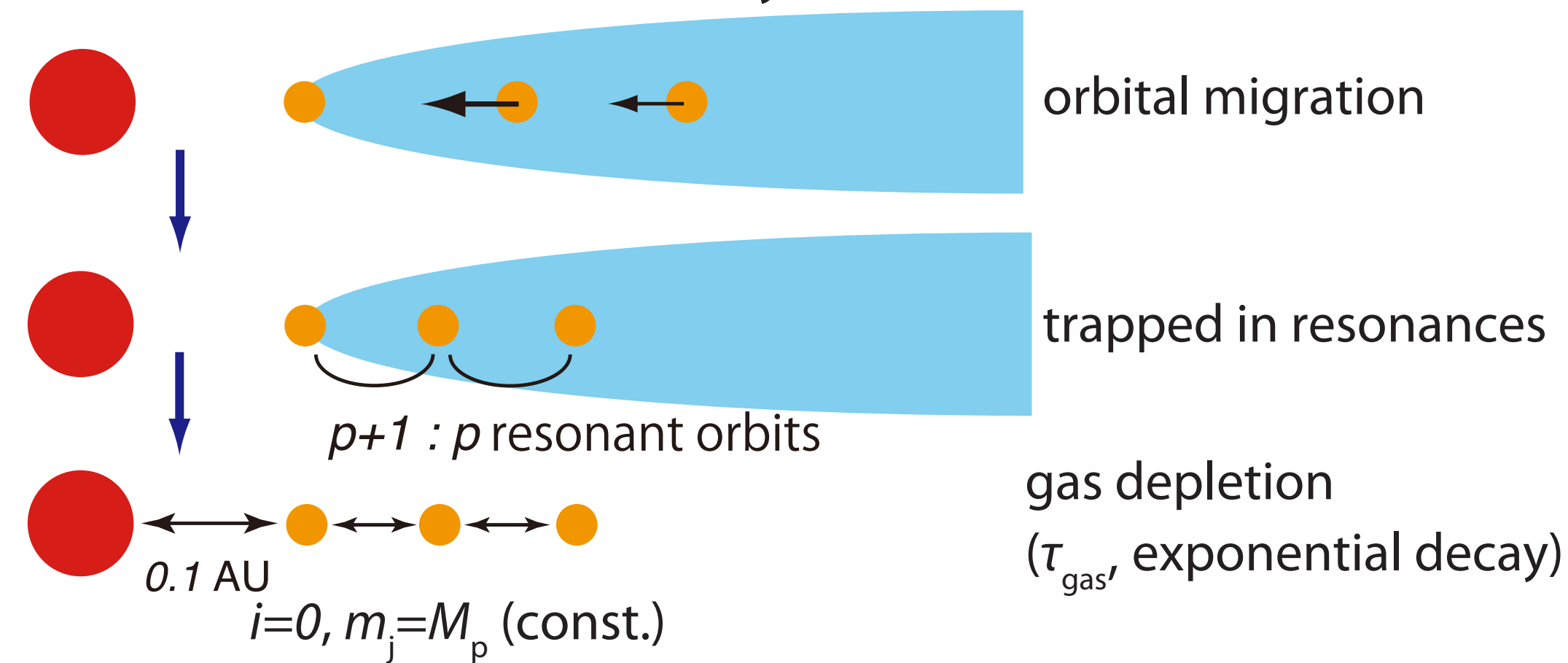
N-body simulations (Terquem & Papaloizou 2007, Ogiwara & Ida 2009)



motivation : We investigated the orbital stability of resonant planets to clarify what makes resonant planets unstable.

Numerical Model

Formation of Resonant Systems



Basic Equation

$$\frac{d^2 \mathbf{r}_i}{dt^2} = -GM_* \frac{\mathbf{r}_i}{r_i^3} - \sum_{j \neq i} Gm_j \frac{\mathbf{r}_{ij}}{r_{ij}^3} - \sum_j Gm_j \frac{\mathbf{r}_j}{r_j^3} + \mathbf{F}_{\text{damp}} + \mathbf{F}_{\text{mig}}$$

(4th Order Hermite Scheme)
gravitational drag type-I migration (Tanaka & Ward 2004, Tanaka+ 2002)

Criterion of Instability

$r_{ij} < r_{\text{Hij}}$ or $\sim 10^8 T_{\text{Kep}}$ (Chambers + 1996),
 $r_{\text{Hij}} = \left(\frac{M_i + M_j}{3M_*} \right)^{1/3} \left(\frac{M_i a_i + M_j a_j}{M_i + M_j} \right)$

Orbital Separation (K)

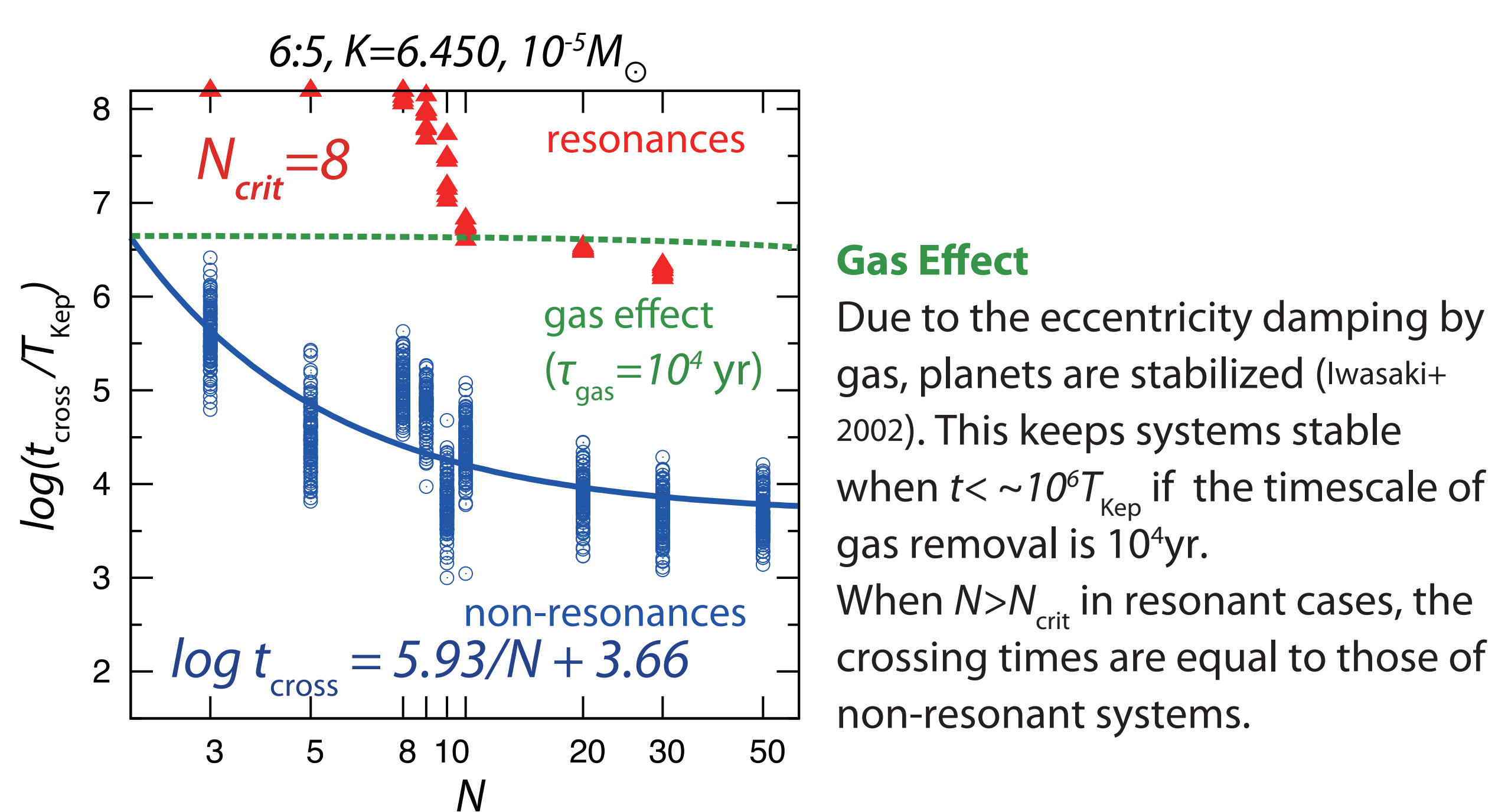
When planets are equal mass and in the same $p+1 : p$ resonances, the orbital separations normalized mutual Hill space are the same, K.

Planetary mass (M_p)

While semi-major axes of $p+1 : p$ resonances are not dependent on M_p , K is dependent on planetary mass due to mutual Hill radius.

Result

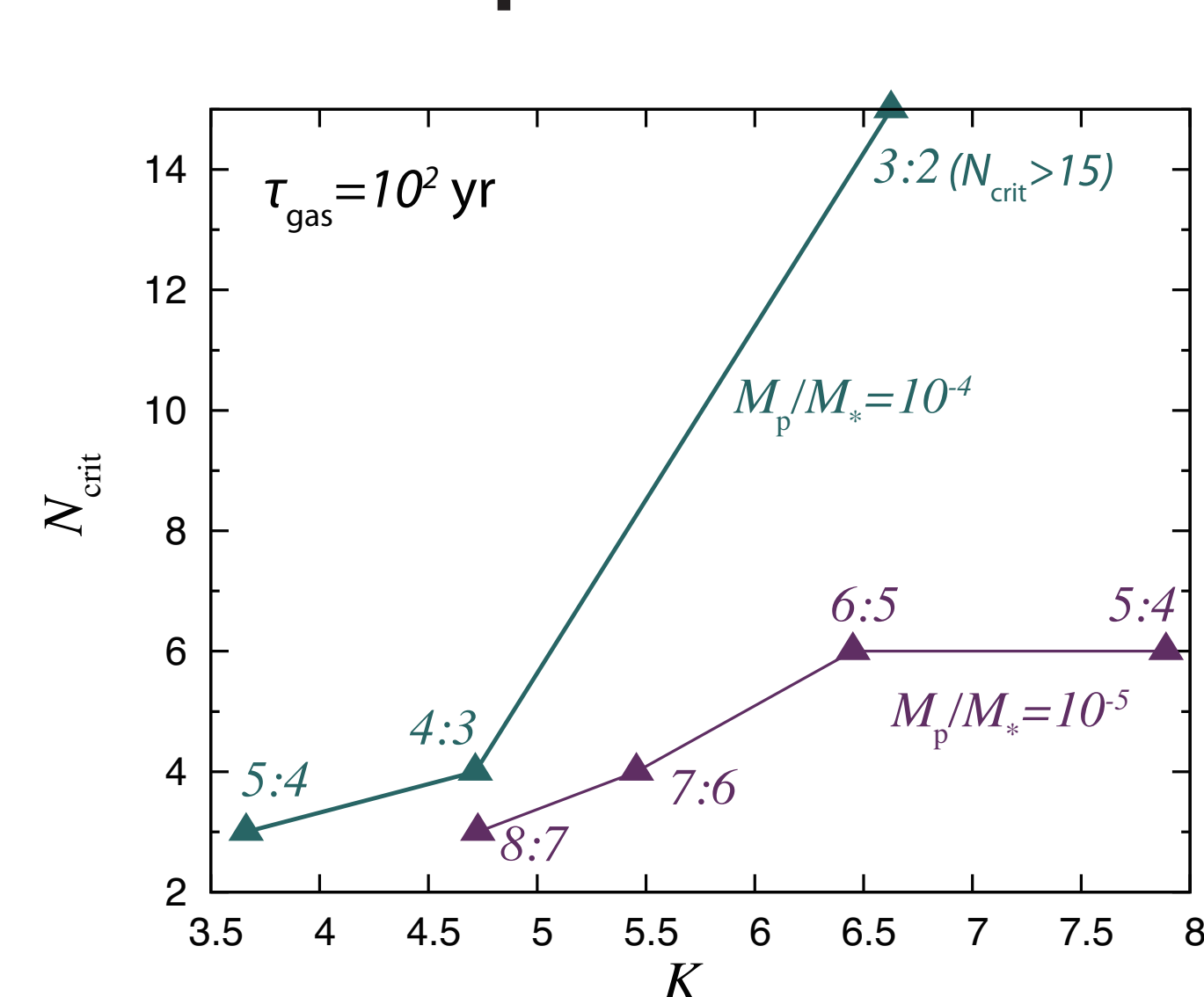
Number of Planets



The crossing times of non-resonant planets is continuously decreasing with N. ($\log t_{\text{cross}} \propto 1/N$)

When $N > N_{\text{crit}}$ in resonant cases, the crossing time decreases rapidly.

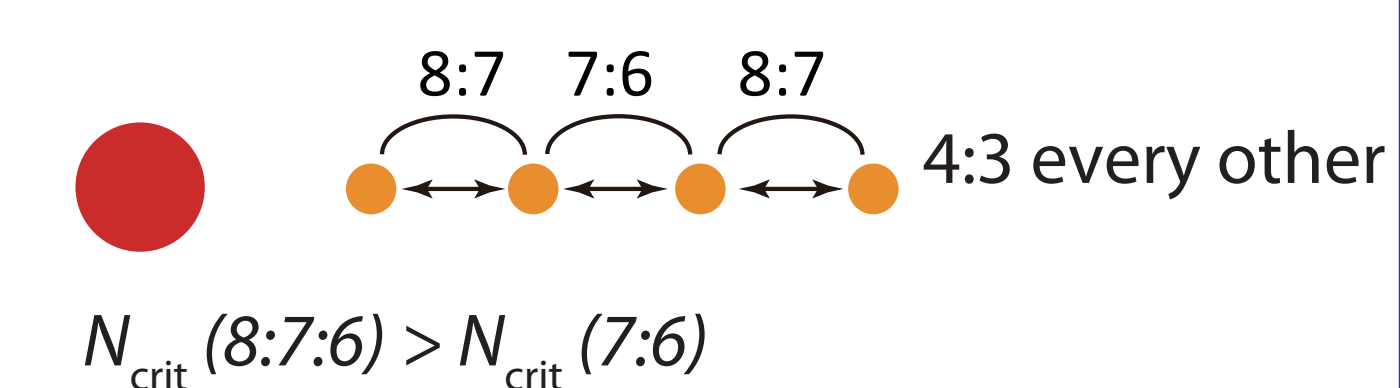
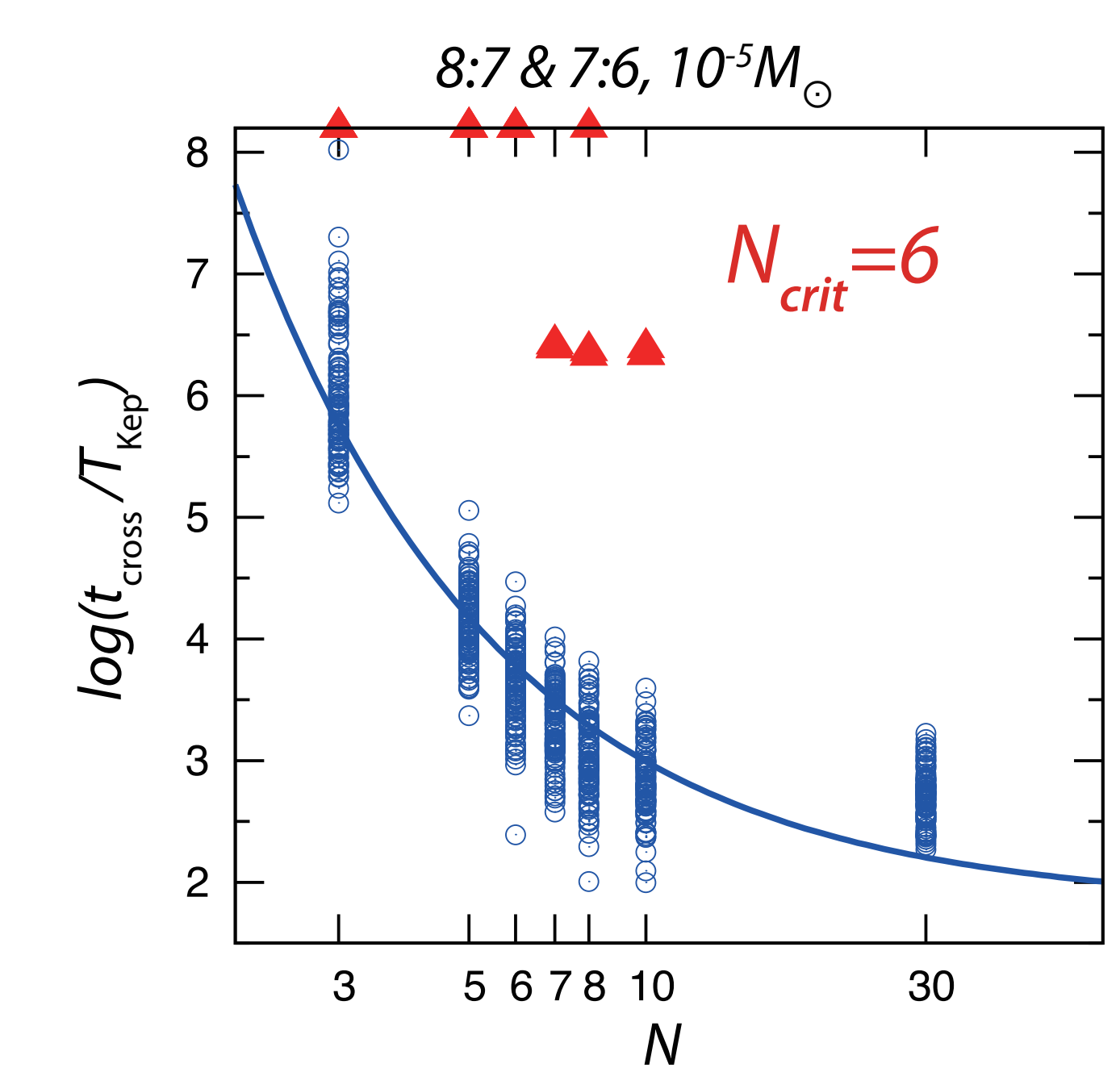
Orbital Separation & Planetary Mass



The larger orbital separation or planetary mass is the longer crossing time is (non-resonance) the larger N_{crit} is (resonance).

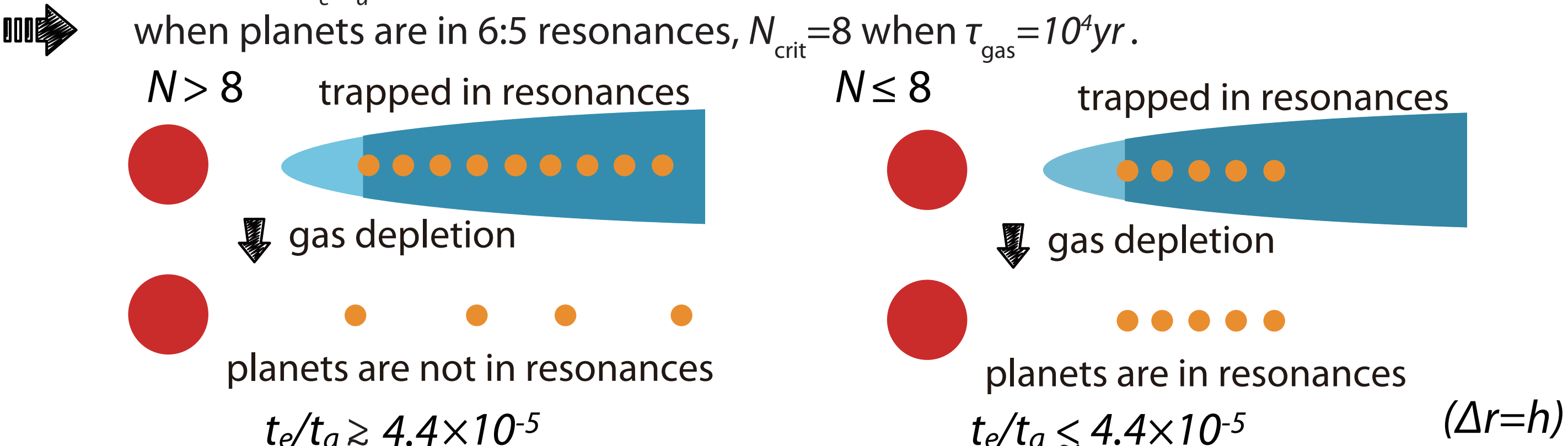
N_{crit} increase with decreasing p ($p+1:p$) when M_p is the same. The resonant overlap (Quillen 2011) would affect N_{crit} .

Chain of Resonances



Discussion

- the number of planets determines whether planets are finally in resonances or not.
- planets also stop migration at the inner edge and the positive migration torque points the positive torque points (e.g., dead zone inner edge, gap by a gas planet, ...) (Morbidelli+ 2008, Matsumoto+ in prep)



Summary

- Although most of observed planets are not in resonant orbits, the orbital distribution of planets has some peaks in or near resonant orbits.
- We investigated the stability time of planets trapped in mean-motion resonances due to orbital migration by N-body simulations.
- the stability time of resonant systems is longer than that of non-resonant system.
- when $N > N_{\text{crit}}$ the planets in resonances become unstable.
- N_{crit} increases with increasing K or increasing M_p .
- N_{crit} of planets in the chain of resonances are larger than N_{crit} of planets in the same resonances.
- Observed systems outside of commensurate orbits would contain the planets over the critical number and these planets cause orbital instability after the gas depletion.