Turbulence Induced Dust Collision Velocities and Rates

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PROBLEM:

Early dust growth in protoplanetary disks occurs collisionally, and, for a broad range of dust sizes, is driven by turbulent flows. The rate of growth, and the final dust grain size distribution, depend sensitively on the velocities and rates of these collisions.

Estimating the velocity scale at which turbulence induced collisions occur is straightforward, but inadequate to the task: using a single characteristic velocity will result in a bouncing-barrier size at which dust grain collisions all result in bouncing. In reality, there is a range of collision velocities, and dust grains that collide at the low end can stick and grow (see Windmark 2012).

We analyzed the problem numerically, and generated **analytical fits for inclusion in future dust coaggulation studies.**

EQUATIONS:

Particles are entrained by gas drag with stopping time τ_p:

$$\frac{\partial \boldsymbol{u}_p(t)}{\partial t} = -\frac{\boldsymbol{u}_p(t) - \boldsymbol{V}(\boldsymbol{x}_p, t)}{\tau_p}$$

τ_p ← Stopping time

To enable our study of dust grain collisions in turbulence, we used artificial turbulence (velocity equation below), which side-stepped the resolution limitations.

$$\boldsymbol{V}(\boldsymbol{x}, t) = \sum_{m,n=0,1}^{m,n=8,3} \sqrt{2} a_{mn}(t) v_m \hat{v}_{mn} \cos\left[\boldsymbol{k}_{mn} \cdot \boldsymbol{x} + \phi_{mn}(t)\right]$$

If interested, ask me what the terms above mean and why they were chosen.

POINT PARTICLES:

As a further complication, dust grains in protoplanetary disks are tiny compared with the turbulent motions. They must be treated as point particles. **To be able to make statements** about the turbulence induced collisions we need to be able to study the turbulence induced collision parameters in the limit of dust grain separation going to zero.

INERTIAL RANGE:

Dust grains for which bouncing is expected to be important are too small/large to care about the largest/smallest scales of the turbulence: one needs a large inertial range, which is out of reach.

IDENTICAL STOPPING TIMES (Hubbard 2012)

Dust grains with identical stopping times are highly correlated.

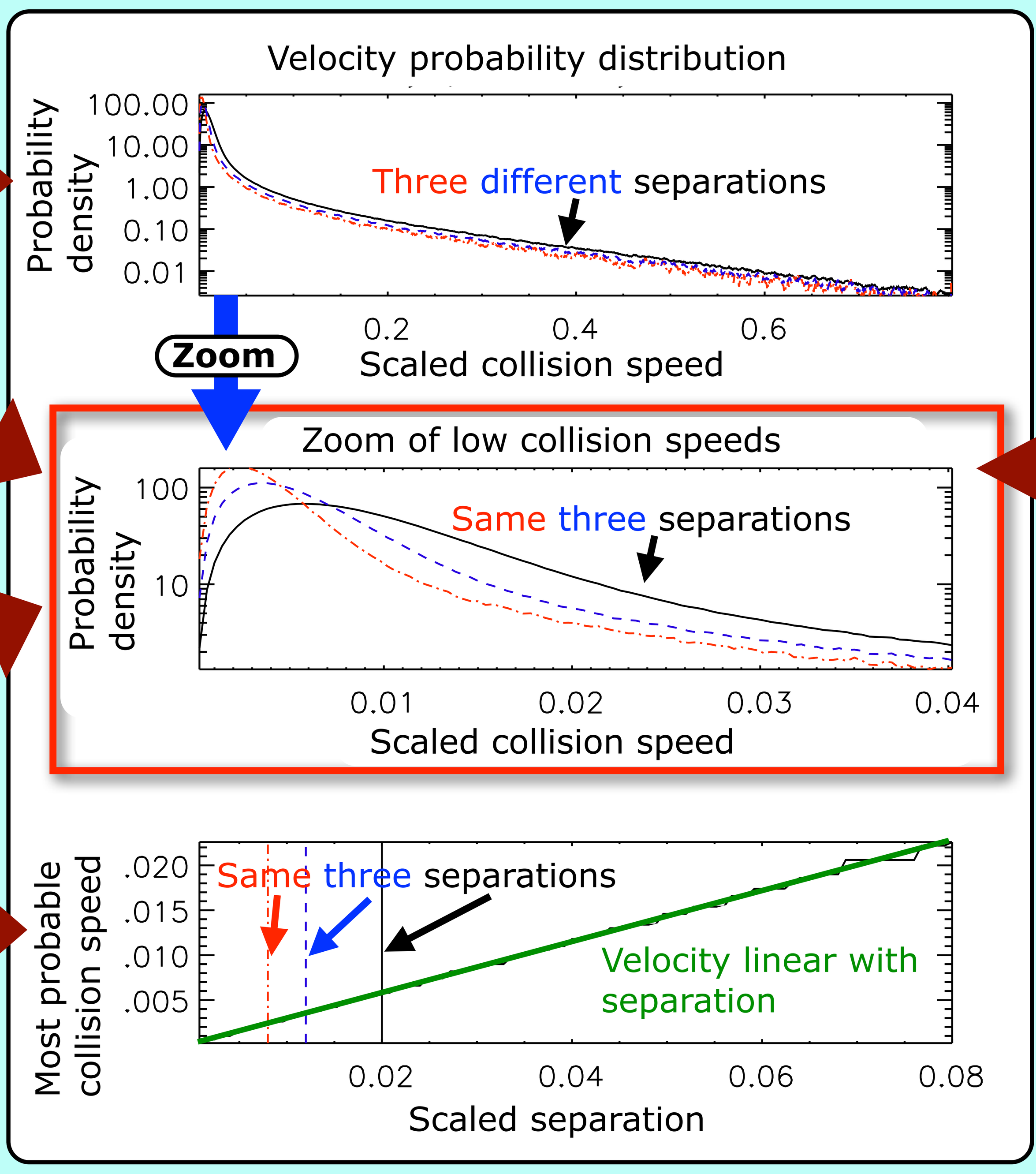
High velocity collision statistics depend only weakly on particle separation.

Low velocity collision statistics depend extremely strongly on particle separation.

We confirm very strong clustering of particles at small separations.

BUT the clusters have very small internal velocity dispersions which are linear in cluster size.

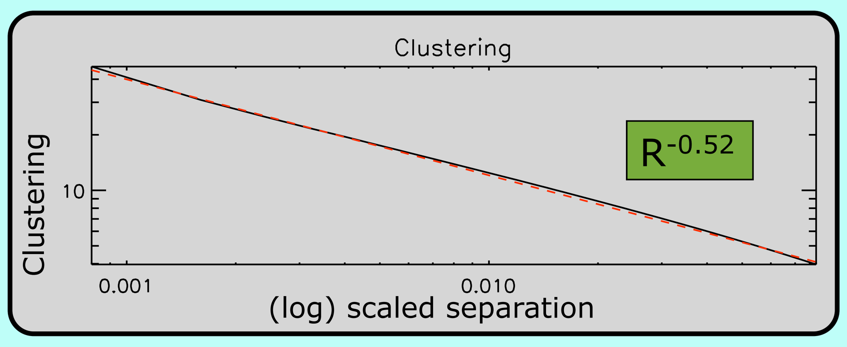
That means that there are no intra-cluster dust collisions. Clusters can still collide with other clusters.



DIFFERENT STOPPING TIMES (Hubbard 2013)

We measure the stopping time ratio with ε. Note that the mass ratio is $(\epsilon+1)^3$.

$$\epsilon \equiv \frac{\tau_1}{\tau_2} - 1 \geq 0$$

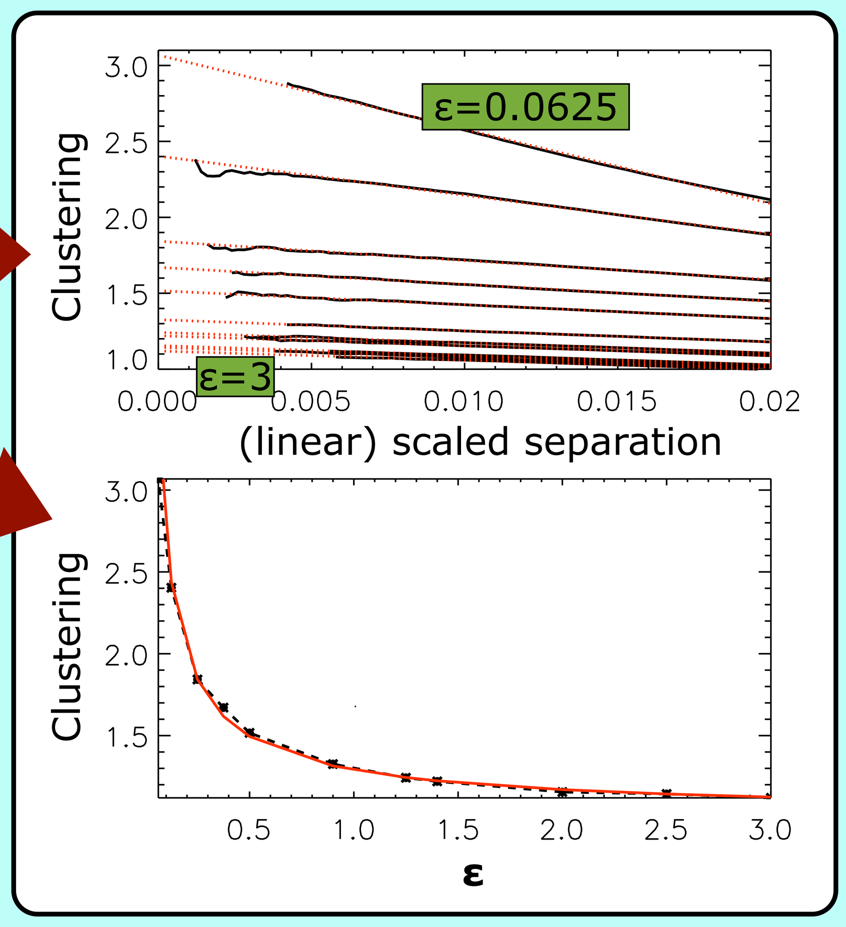


Dust grains with *identical* stopping times see *divergent* clustering.

By comparison, particles with *different* stopping times see *finite* clustering.

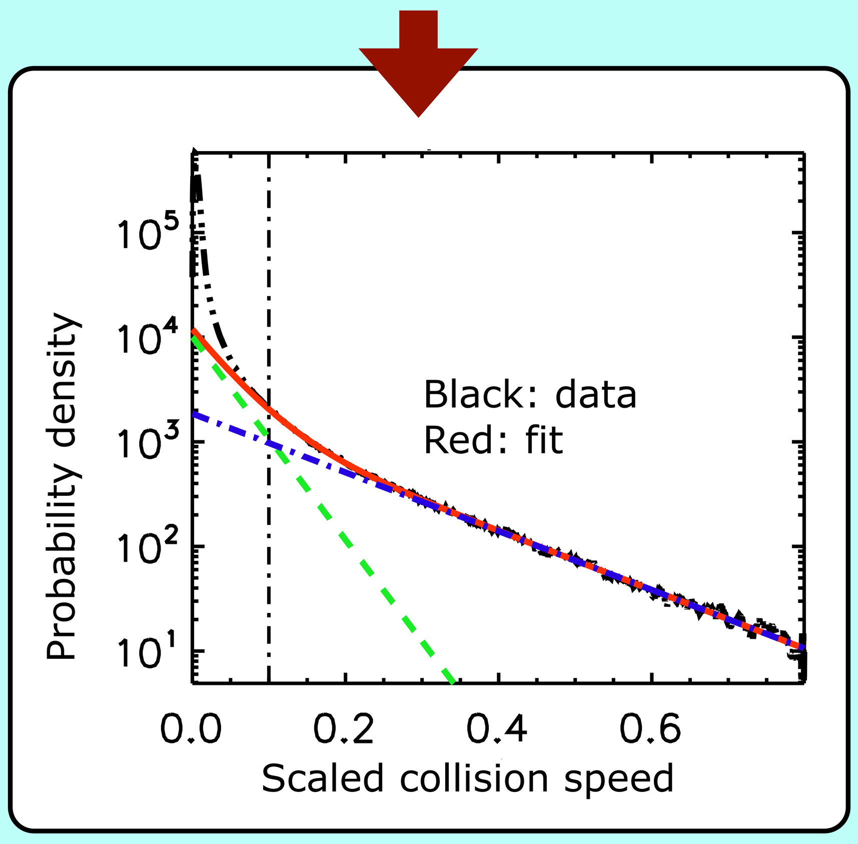
Clustering is stronger for particles with more similar stopping times.

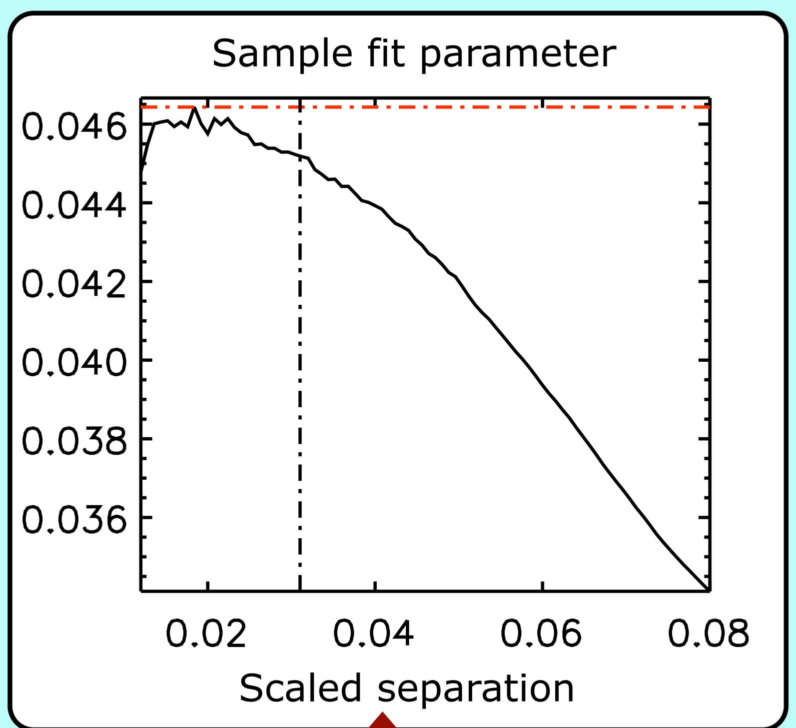
Unless the dust size distribution is very tightly peaked, turbulent clustering is unlikely to create dense enough clusters to trigger the streaming instability of Johansen et. al. 2007.



Fitting collisional velocity probability distributions: identical stopping times (see Hubbard 2012 for details)

Except at very low collisional velocities, the collisional velocity probability distribution for particle pairs with identical stopping times is well fit by the sum of two exponentials:

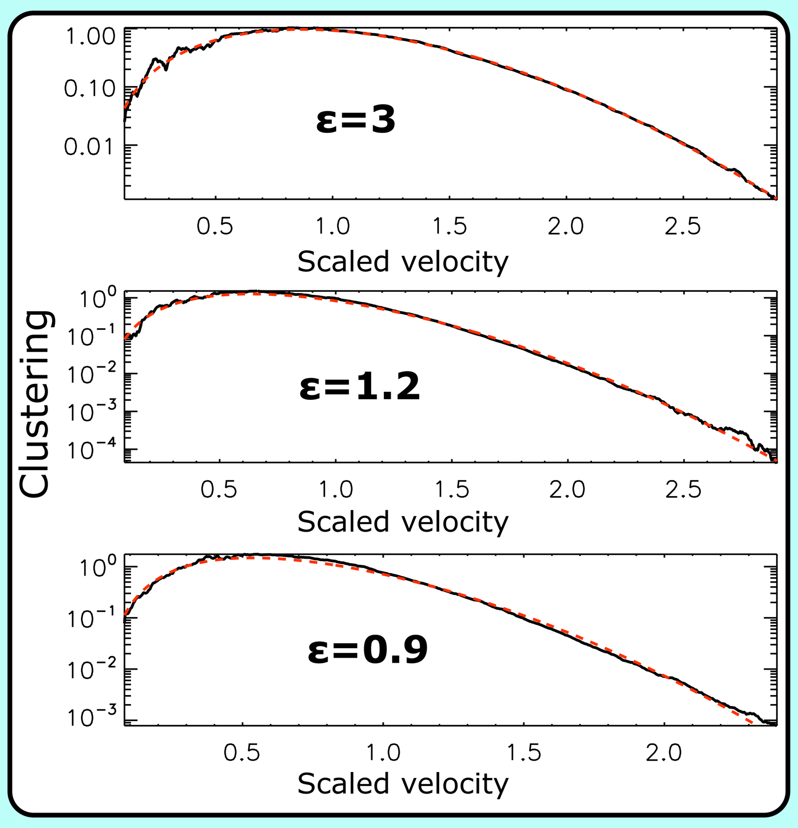


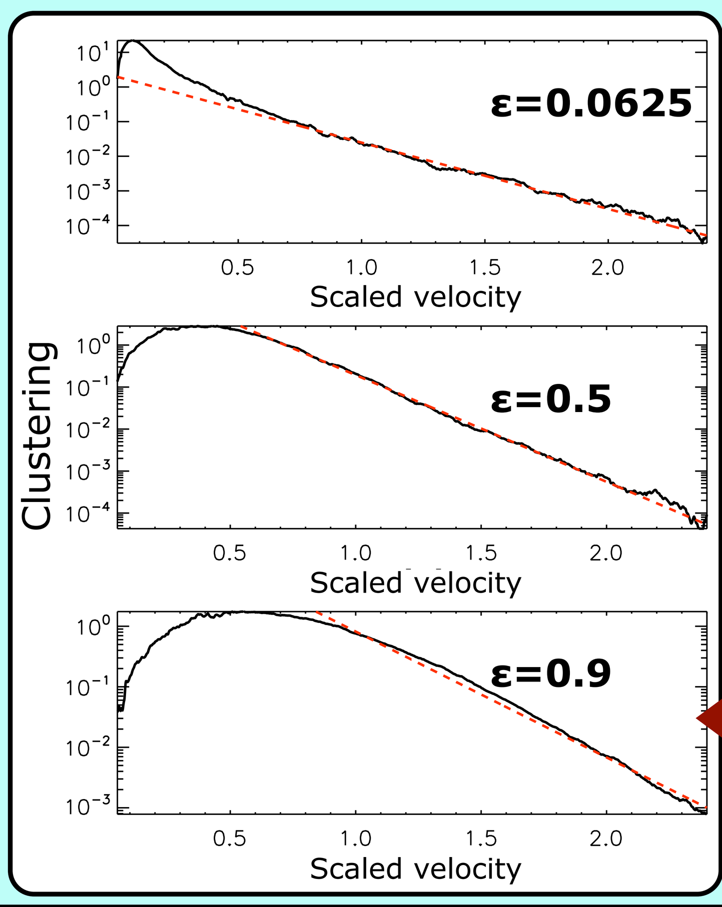


As shown above for one of the fit parameters, the parameters reach a steady limit as the separation of the dust grains approaches 0: the results hold for point particles.

Fitting collisional velocity probability distributions Different stopping times (see Hubbard 2013 for details)

For ε>0.9 (about a 10-fold mass difference) the collisional velocity probability distribution is nearly Maxwellian.





For ε<0.9 (about a 10-fold mass difference) the collisional velocity probability distribution is **not** Maxwellian.