Reaccretion Efficiencies in Small Impactor - Large Target Collisions

UNIVERSITÄT DUISBURG ESSEN
Open-Minded

Tim Jankowski[1]*, Gerhard Wurm[1], Jens Teiser[1]

[1]University Duisburg-Essen, Faculty of Physics *tim.jankowski@uni-due.de

July 15-20, 2013
Heidelberg
Protostars & Planets VI

Introduction

During the formation process of planets in protoplanetary disks small dust particles grow to km sized planetesimals and beyond via collisions. During the different stages, collisions occur not only between equal sized bodys.
While the gas in protoplanetary disks exerts size- and mass-dependent drag forces on the dust particles and bodys present, the relative velocities between the small particles and larger bodys increase.
A field of investigation are the small-impactor large-target collisions where (partial) erosion can occur and small ejected dust particles can be produced. These ejecta can couple to the gas quite rapidly and can then be recaptured by the target and stick to it in secondary collisions [1,2] (fig. 1).
We use a Monte-Carlo code to calculate reaccretion efficiencies under certain conditions i.e. in free molecular flow regime. Experimental data was used to develop models for the quantity of ejecta, the ejecta velocities and ejecta directions.

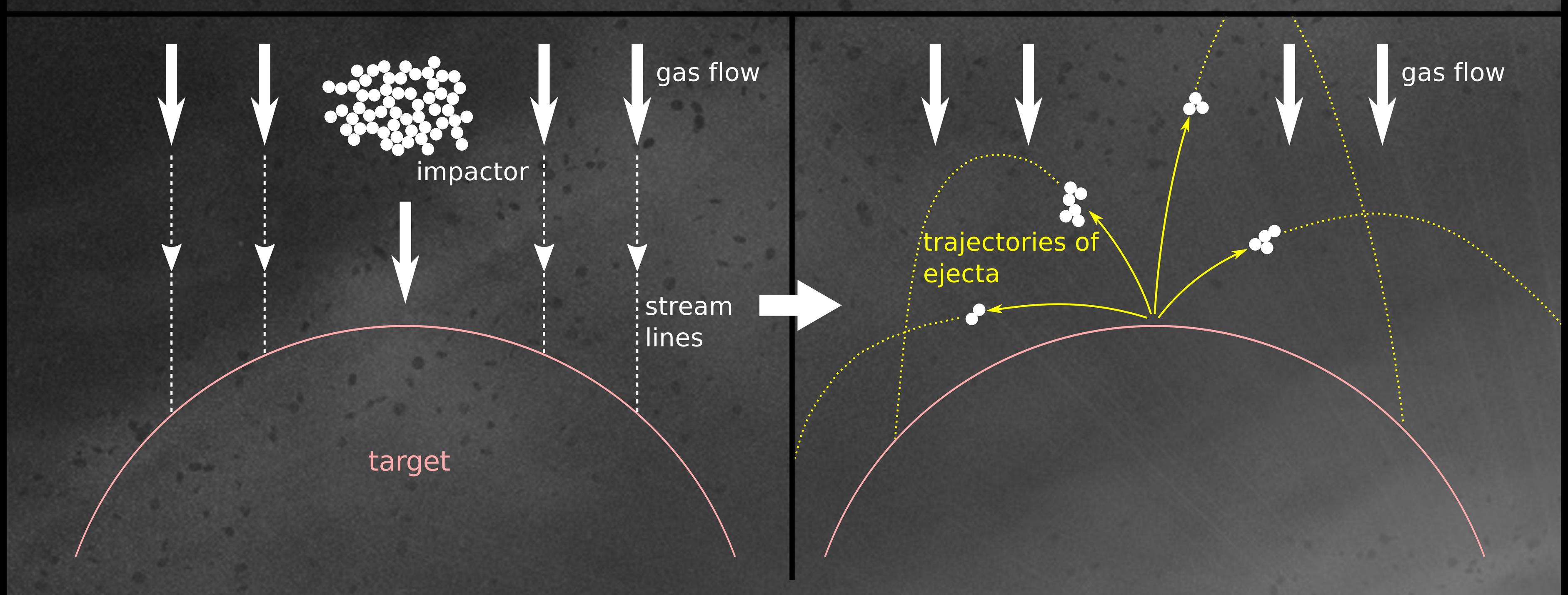

FIG. 1. Small impacting particles can lead to partial fragmentation and to even smaller ejected particles. These ejecta were influenced by the gas motion (low gas-grain coupling time, see right) and can be accreted by the target in secondary collisions ("reaccreted"). The impact-velocity accords to the gas-velocity in respect to the target body.

What we have done

Using a Monte-Carlo code we simulated the ejection of fragments in small impactor - large target collisions focussing on how many of them can be recaptured by the target in secondary collisions (see figure on the right).

We found that the amount of reaccreted particles is highly dependent on a lot of parameters but generally benefits from high impact velocities (= high gas velocities) and a large size difference between target and impactor (fig. 7 - 9).

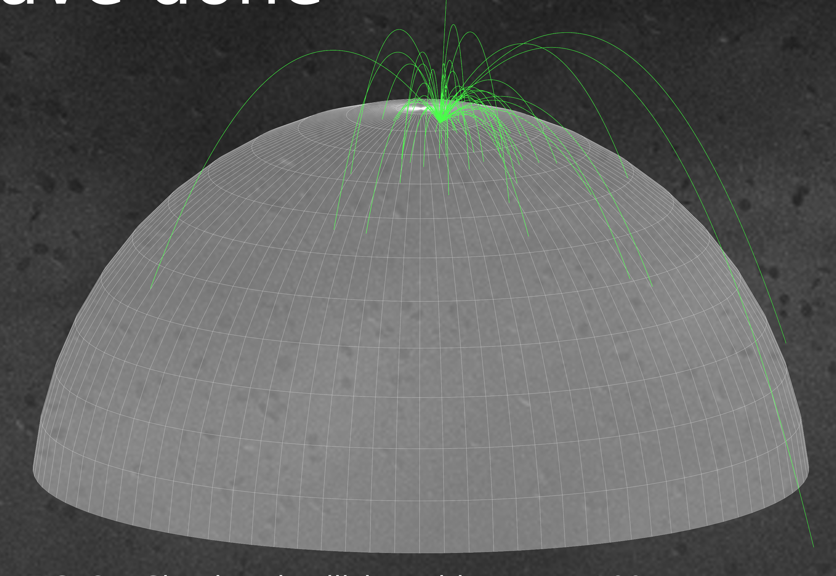

FIG. 2. Simulated collision with r_{target} = 100 mm, $r_{impactor}$ = 0.2 mm, $v_{impactor}$ = 5 m/s, T_{gas} = 300 K and p = 0.01 mbar. Within this collision, only 3 of the more than 50 ejecta do not hit the target in secondary collisions.

Motion of Particles in Gas

When particles are situated in gaseous environments, the motion of these is adapted to the motion of the gas in specific timescales (Epstein regime). Therefore as long as $v_{gas} \neq v_{particle}$ a force is acting, which can be described as

$$F = \frac{m}{\tau_f} v_{gas} \quad \text{, with} \quad \tau_f = \epsilon \frac{m}{\sigma_a} \frac{1}{\rho_g v_m}$$

and m is the particle mass, v_{gas} is the gas velocity in respect to the particle, τ_f is the gas-grain coupling time [3], σ_a is the geometrical cross section of the particle, ρ_g is the gas density. v_m is the mean thermal velocity of the gas molecules, and ϵ = 0.58 is a numerical factor.
Using the solution for the equation of motion, the movement of an ejecta can be analytically determined to

$$\mathbf{r}(t) = \mathbf{r}_{init} + [\mathbf{v}_{init} + \mathbf{v}_{gas}]\, \tau_f \left(1 - \exp\left(-\frac{t}{\tau_f}\right)\right) ,$$

where $\mathbf{r}_{init}$ is the initial position, $\mathbf{v}_{init}$ is the initial particle velocity and $\mathbf{v}_{gas}$ is the gas velocity (see fig. 3).

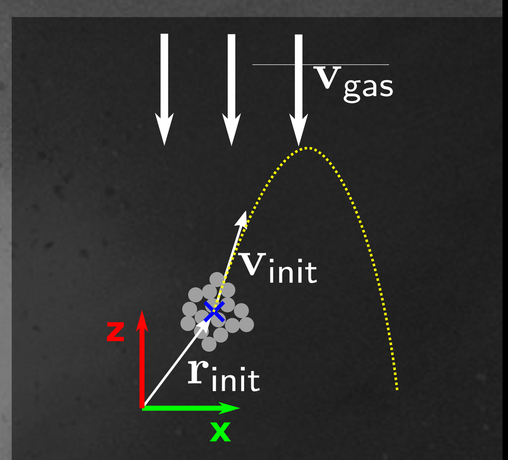

FIG. 3. Scheme of a particle in gas flow with resulting trajectory (yellow dotted), calculated from the formula described left. Note that the resulting trajectory is no parabola but $\mathbf{v}(t) \rightarrow \mathbf{v}_{gas}$ for $t \rightarrow \infty$.

Preliminary Results

For every set of parameters, around 20,000 collisions have to be simulated to get sufficient precision of approx. 0.1 %. Due to the multiplicity of free parameters (impact velocity, impactor size and density, target size and density, gas pressure, and gas temperature) the aim is to determine an analytical function for the reaccretion efficiency in respect to the free parameters.

Reaccretion efficiencies η can either be specified by the ratio of the quantity of reaccreted ejecta to the total quantity ($\eta_\#$) of ejecta or (preferable) by the mass of reaccreted ejecta to the total mass (η_m). Note that in this model every hit in a secondary collision with the target is treated as sticking collision.

Preliminary results for η with different sets of free parameters are shown in Fig. 7 to 9.

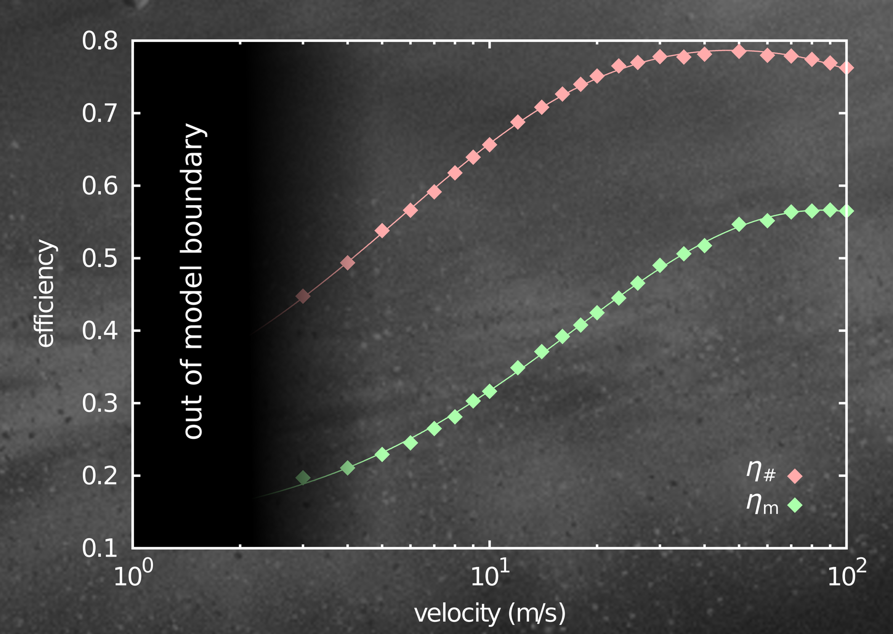

FIG. 7. Reaccretion efficiencies η_m and $\eta_\#$ for collisions at r_{imp} = 0.2 mm, r_{target} = 400 mm, T = 300 K, p = 0.01 mbar. The fit functions have the form

$$-a + \frac{(a+b)(x+c)}{\sqrt{1+(x+c)^2}} - d \cdot (x+c) \; .$$

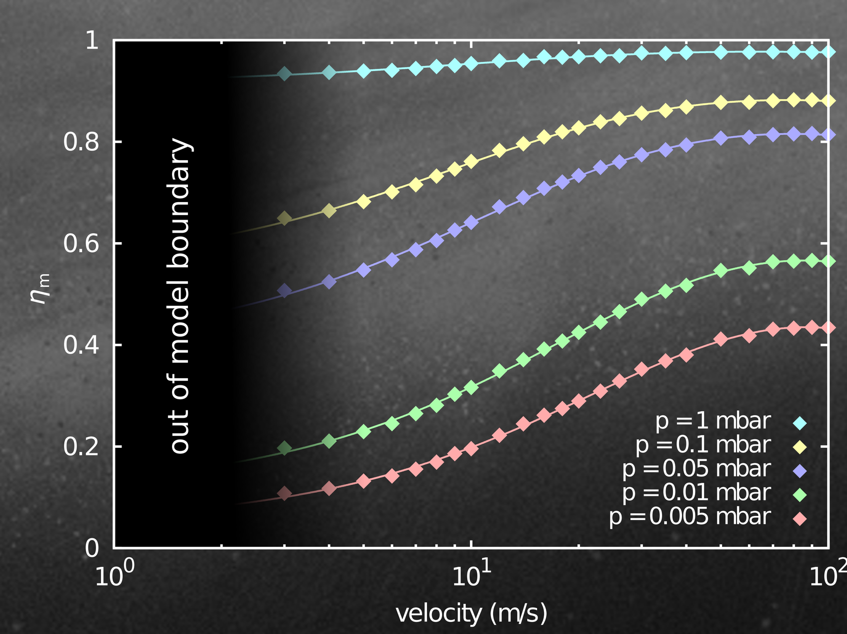

FIG. 8. Reaccretion efficiency η_m for collisions at r_{imp} = 0.2 mm, r_{target} = 400 mm, T = 300 K, and different pressures. The fit functions have the form

$$-a + \frac{(a+b)(x+c)}{\sqrt{1+(x+c)^2}} - d \cdot (x+c) \; .$$

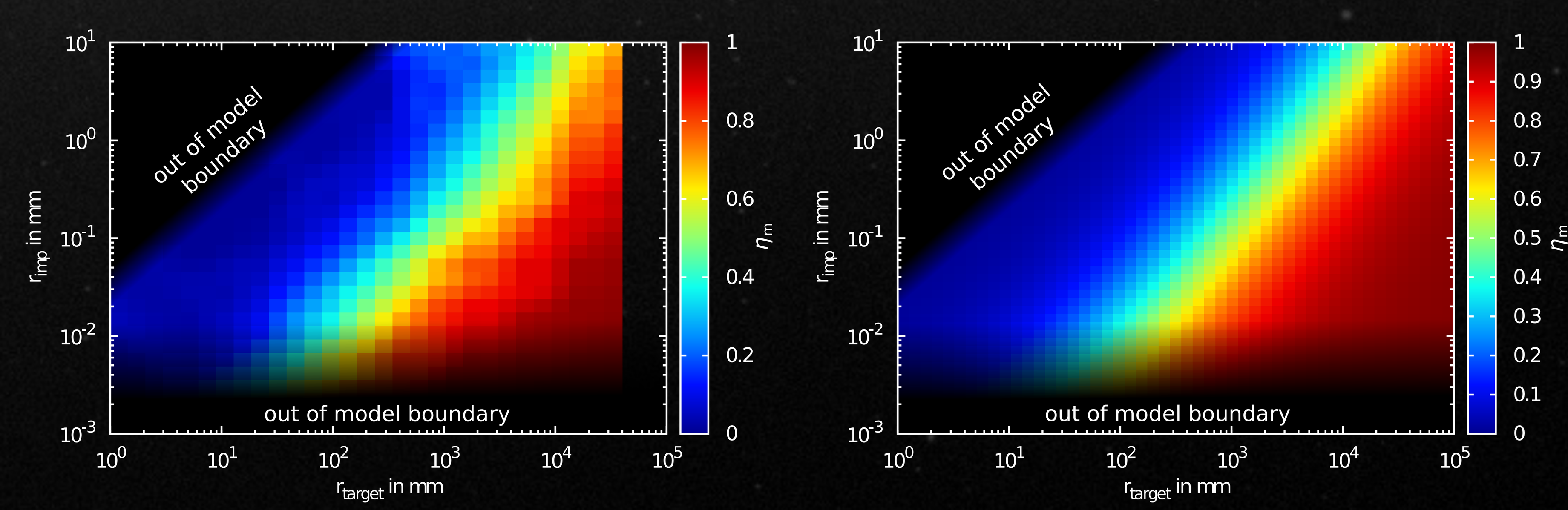

FIG. 9. Reaccretion efficiencies η_m for collisions at v_{imp} = 5 m/s, T_{gas} = 300 K, p = 0.01 mbar.
Left: interpolated data (more than 250 datapoints are used)
Right: fitted distribution of the data. As fit function a logarithmic S-curve with y ∈ [0:1] was used:

$$\eta_m(r_{target}, r_{imp}) = \frac{1}{2}\tanh\left(- b(x) \cdot \log(y) - a(x) \right) + \frac{1}{2}$$
$$= \frac{1}{2}\tanh\left(-(b0 - b1 \cdot \log(x)) \cdot \log(y) - (a0 - a1 \cdot \log(x))\right) + \frac{1}{2}$$

Simulation and Model Details

The approach to simulate reaccretion is as follows:

1. set start variables:
mass of impactor, gas velocity and position of impact (in respect to model boundaries)

2. use model to determine variables:
quantity of ejecta, ejecta velocities, masses, and directions, model based on experiments

3. calculate trajectories
use numerical approach to check if ejecta hit the target and identify the impact position

From experimental data [4,5,6] we developed a probability function for the mass of an ejecta depending on the impactor mass and impactor velocity using a Weibull distribution.
By dint of a random number generator (Mersenne Twister), we generate ejecta until $\sum m_i = M_{impactor}$.
We created a distribution function for the directions of the ejecta $\mathbf{v}_{ejec}/|\mathbf{v}_{ejec}|$ where the tangential component (represented by the angle γ) is uniformly distributed and the normal component (δ) follows a weibull function (see Fig. 4,5).
To assign velocities ($|\mathbf{v}_{ejec}|$) to the ejecta, we used the data to create a probability function subject to the impactor velocity.
To prevent underestimation of the fact that the impacts can be very skew, the tangential component of $\mathbf{v}_{imp}$ is added to $\mathbf{v}_{ejec}$ modified by a factor $\sin(\vartheta) \cdot \mathbf{v}_{imp}/\mathbf{v}_{ejec}$.

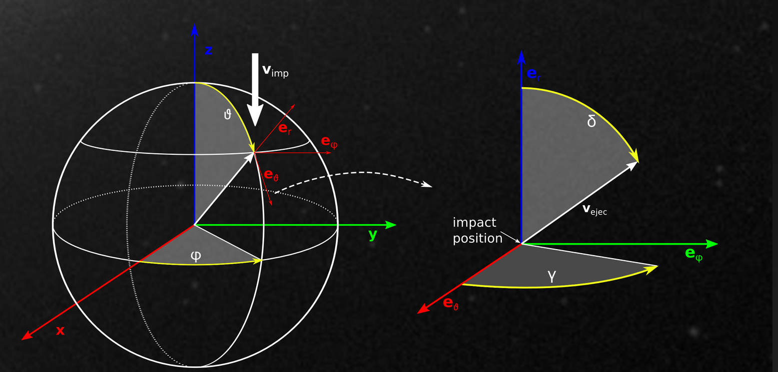

FIG. 4. Left: Impact position in respect to the spherical target. Right: Visualisation of the ejecta direction in a spherical coordinate system. A uniform distribution is used for γ and a Weibull distribution for δ (see below).

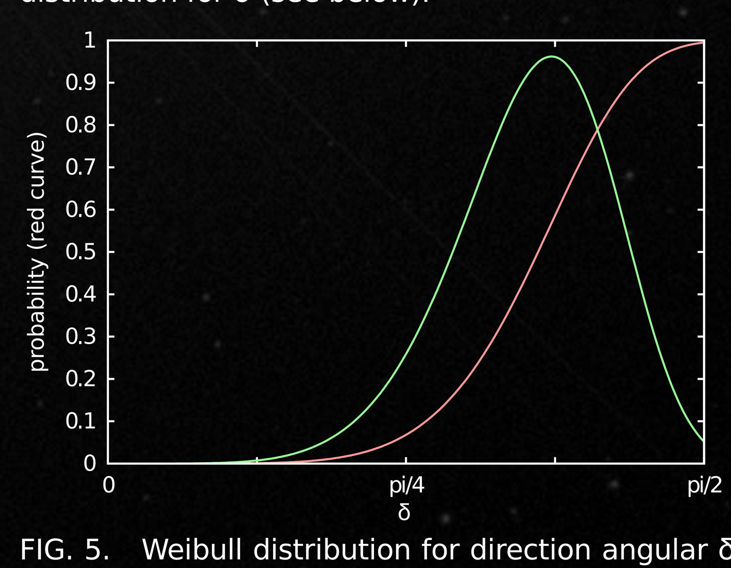

FIG. 5. Weibull distribution for direction angular δ (green, arb. units) and resulting cumulative distribution function (red).

Outlook

Yet there is not sufficient data to determine an analytical expression for η_m in respect to the free parameters. Further simulations are running continuously although the code still has to be improved for faster computation.
Another point of interest is to use models for protoplanetary disks for sweeping the free parameters dependent on the distance to the central star.

References

[1] G. Wurm, J. Blum, and J.E. Colwell
Icarus **151**, 318-321 (2001)
[2] G. Wurm, J. Blum, and J.E. Colwell
Phys. Rev. E **64**, 046301 (2001)
[3] J. Blum et al.
Icarus **124**, 441-451 (1996)
[4] J. Teiser, I. Engelhardt, and G. Wurm
ApJ **742**, 5 (7pp) (2011)
[5] J. Teiser, M. Küpper, and G. Wurm
Icarus **215**, 596-598 (2011)
[6] T. Meisner, et al., in prep.

Special thanks to D. Veberic for providing a fast c++ code to calculate the Lambert W (x) function.

This work is supported by the DFG.