Circumbinary Disk around Eccentric Binary

Yusuke IMAEDA (Tokyo Tech)

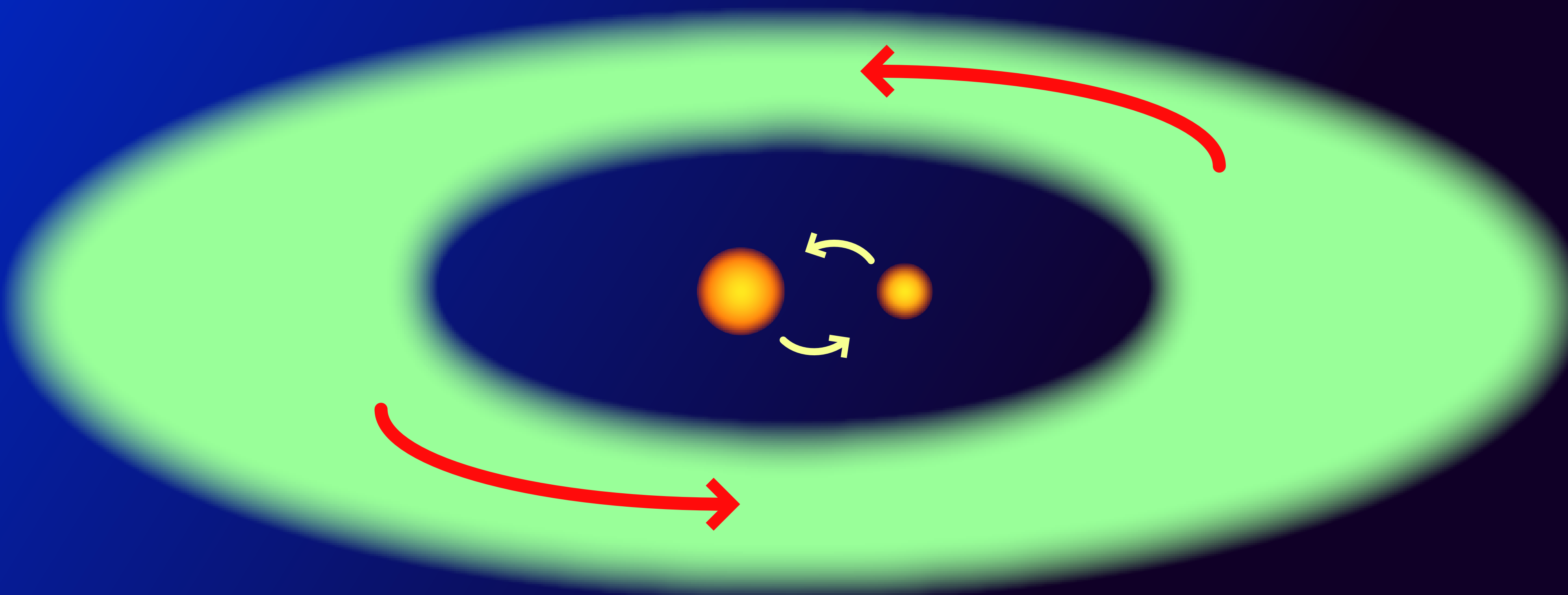
Most stars are born as a binary system or a multiple system. Usually, its orbit is eccentric and the binary is tidally interact with the surrounding gas disk for a long time. We present the numerical results of the long-term interaction between the eccentric binary and the circumbinary disk.

Instead of a direct calculation of the fluid equation, we solved a time averaged evolution equation for the gas orbital element. The assumption we made is that the change of orbital parameter for the gas element is much longer than the orbital period of the corresponding semimajor axis. The dominant motion of the gas disk is Keplerian rotation around the mass center. The gravity force deviation and the pressure force are much smaller than the dominant gravity. Thus, the above assumption is valid. This indicates that the orbital change in one orbital period can be neglected and the gas disk can be treated as an assembly of gas rings. Time averaged evolution equation for the gas orbital element is calculated over these gas rings. The results shows the eccentric binary makes the circumbinary disk also eccentric. Some eccentricity and pericenter gradient structure are formed in the disk, which can make the banana-like density structure in the circumbinary disk, if the gradient becomes sufficiently large.

We also calculate the mass accretion from the eccentric circumbinary disk to the eccentric binary stars with Godunov SPH.Various disk eccentricities are considered. The results shows that the mass accretion rate drastically increases if the disk eccentricityis high enough. This indicates the opportunity that rapid gas dissipation occurs when the disk becomes eccentric by the secular evolution around the eccentric binary.

Initial Setup

Gas disk is tidally interact with binary stars for a long time



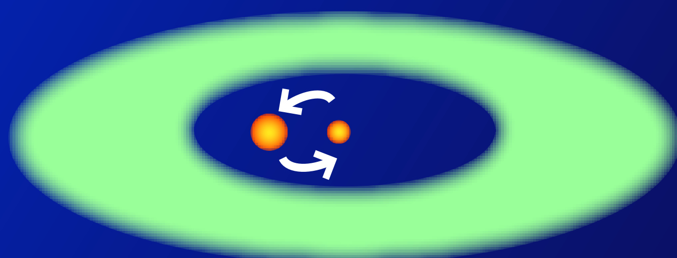
Density structure? Velocity Field? After secular evolution

← Mass Ratio? Eccentricity? Temperature?

Various timescales in the disk

The Long-Term Evolution

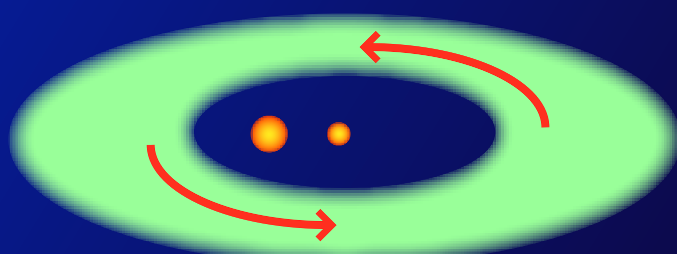
Dynamical time scale of the binary



If the semimajor axis is several tens of AU

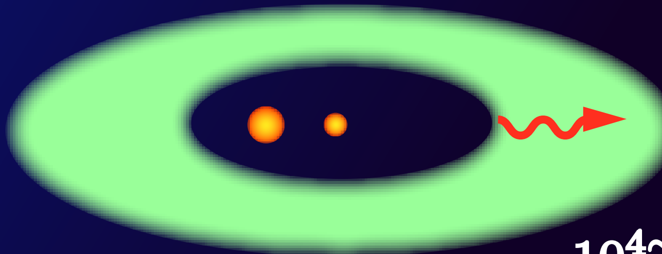
$\sim 10^{2}$ year

Dynamical time scale of the gas disk



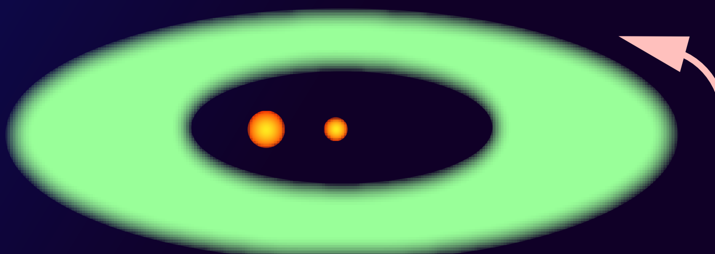
$\sim 10^{3}$ year

Sound crossing time scale



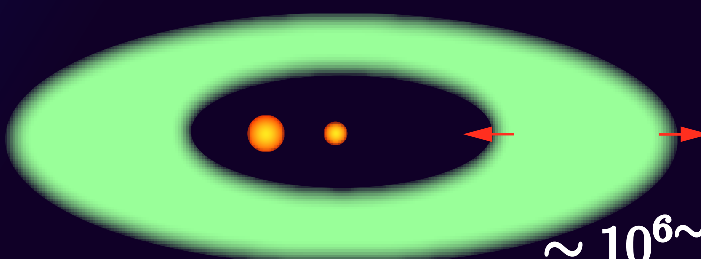
$\sim 10^{4\sim5}$ year

Precession time scale



$\sim 10^{5\sim6}$ year

Viscous time scale

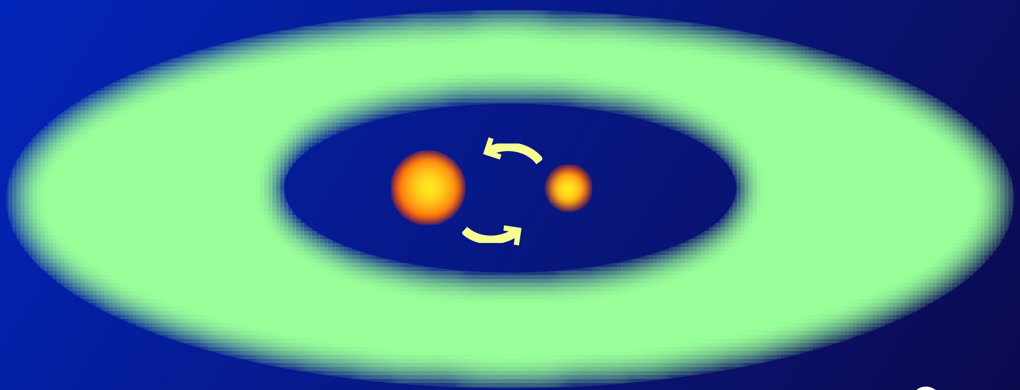


$\sim 10^{6\sim7}$ year

Calculation (2D)

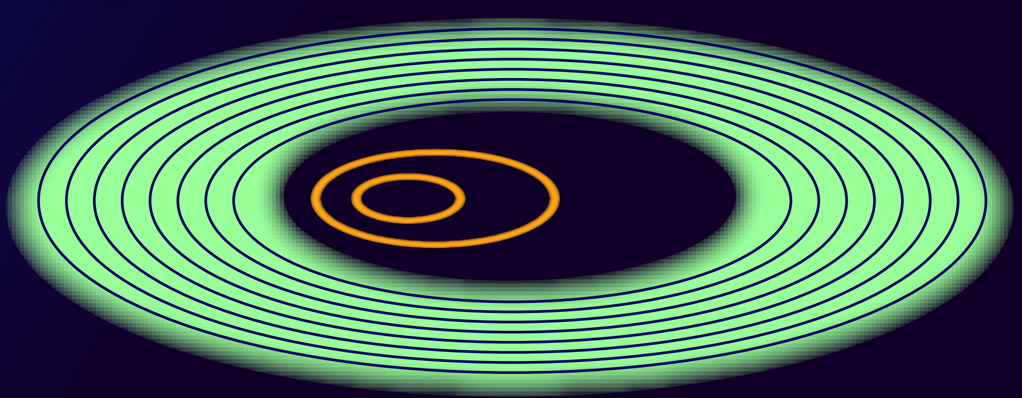
Split the long-term component and the rapid component

$$\frac{dx}{dt} \quad \frac{dy}{dt} \quad \frac{dv_x}{dt} \quad \frac{dv_y}{dt} \longrightarrow \frac{da}{dt} \quad \frac{de}{dt} \quad \frac{d\omega}{dt} \quad \frac{df}{dt}$$



Long-term component a, e, ω

Rapid component f



Averaged by 1 rotation

Binary Gravity → Ring Gravity

Gas Pressure → Ring-Ring Interaction

Secular evolution of orbital elements are writen as follows:

$$\left\langle \frac{da}{dt} \right\rangle = 0$$

$$\frac{1}{i\Omega_B}\left\langle \frac{d\psi}{dt} \right\rangle = \frac{d^2\psi}{d^2\xi} + U\psi + V$$

where $\psi = e\cos\omega + i\, e\sin\omega$

Potential U: axisymmetric perturbation

Force Term V: non-axisymmetric perturbation

From binary model and the disk model → U, V

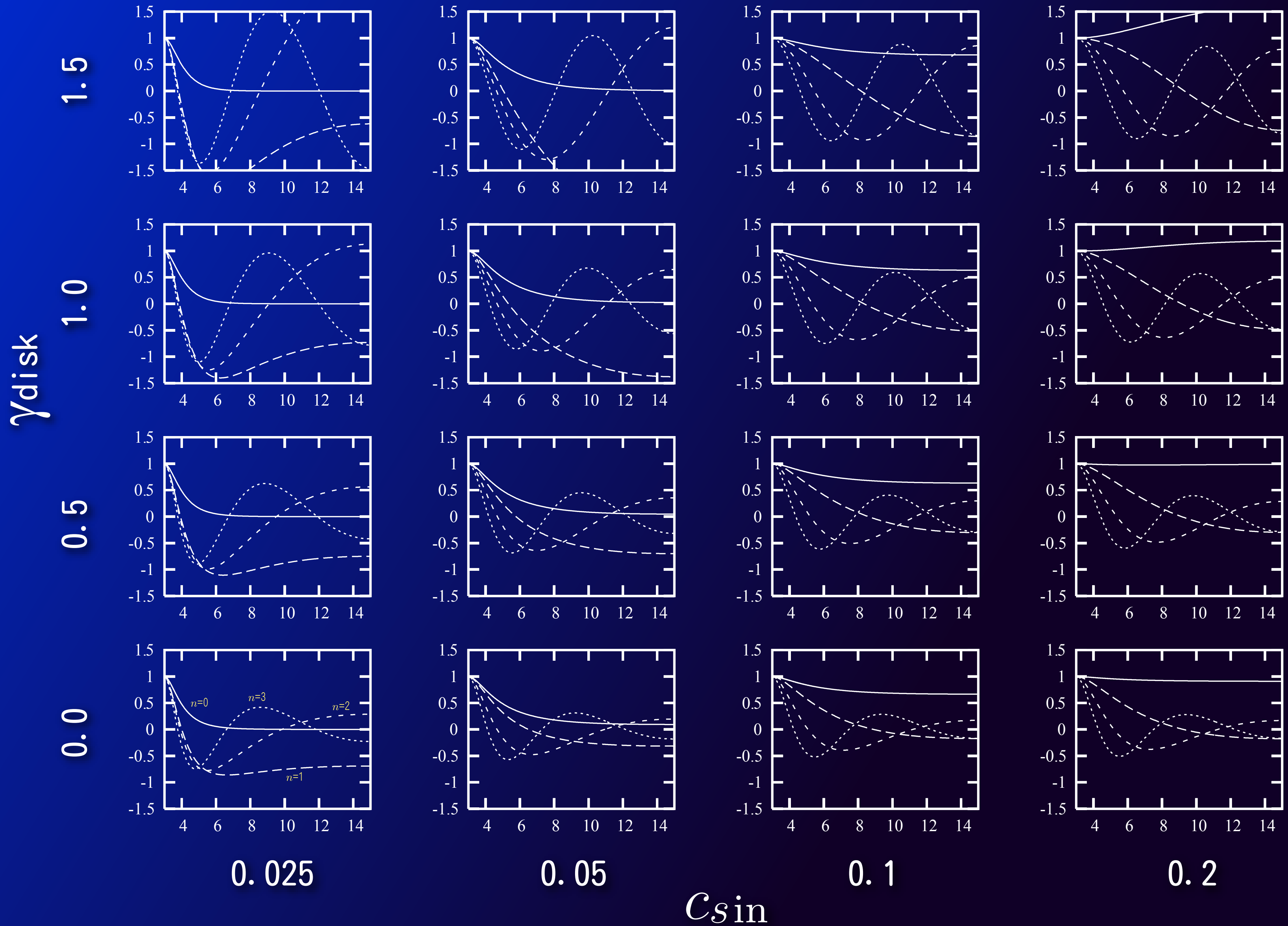
U → Eigen Wave pattern and the pattern speed

V → Amplitude

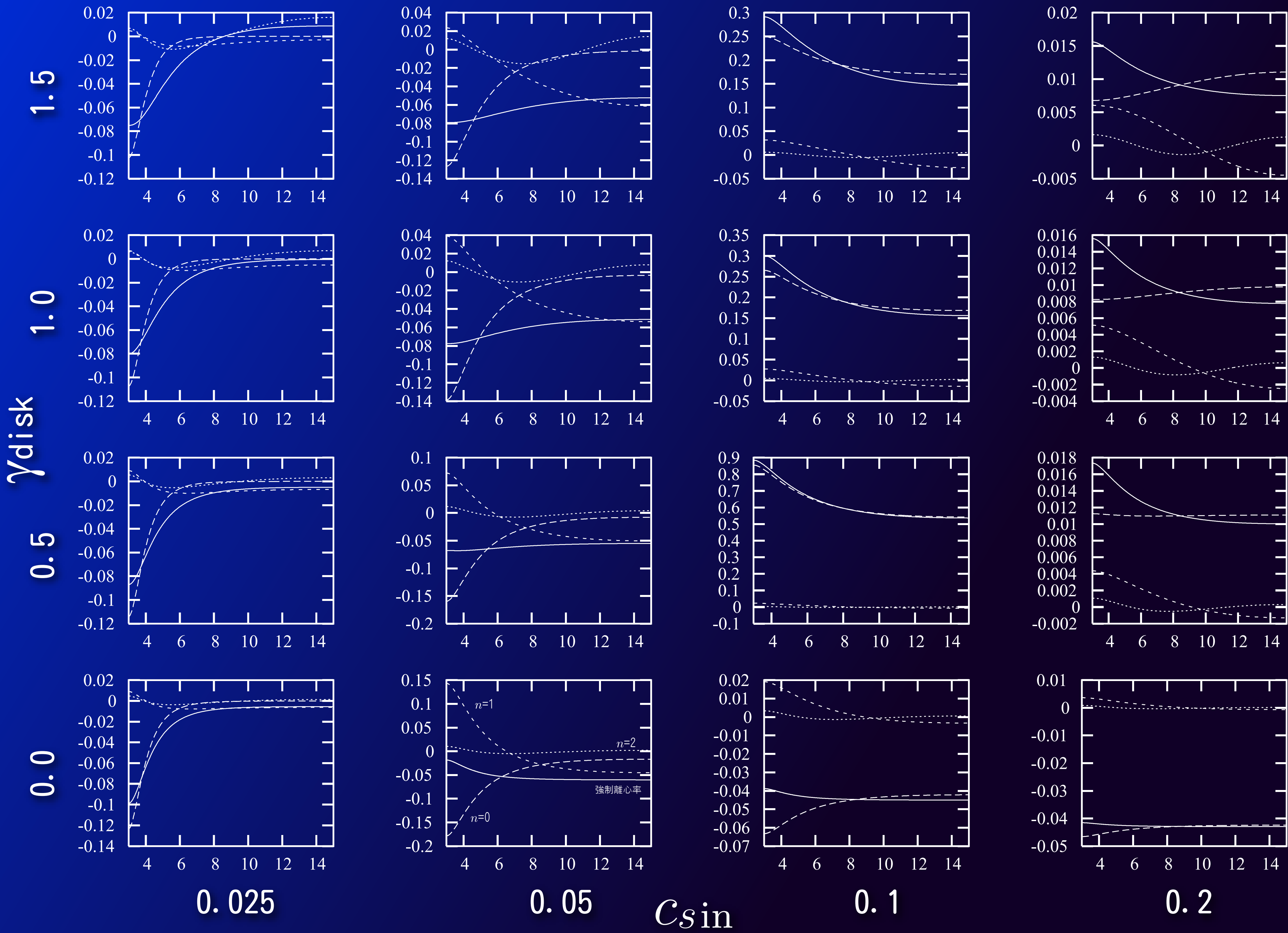
→ **Long-term Evolution of the disk**

$$\begin{aligned}
k &\equiv e\cos\omega \\
h &\equiv e\sin\omega \\
A &\equiv \frac{3}{4}q_1 q_2 \left(\frac{a}{a_B}\right)^{-\frac{7}{2}} \left(1+\frac{3}{2}e_B^2\right)\Omega_B \\
B &\equiv \frac{15}{16}q_1 q_2 (q_1 - q_2) \left(\frac{a}{a_B}\right)^{-\frac{9}{2}} \left(1+\frac{3}{4}e_B^2\right) e_B \Omega_B \\
\xi &= \int \frac{1}{aZ}da \\
\psi &= (k+ih)\exp(-\int X d\xi) \\
Z &= \frac{1}{\sqrt{2}\mathcal{M}_L}\left(\frac{a}{a_B}\right)^{-\frac{3}{4}} \\
X &= -\frac{1}{2}\left[Z\left(3+\gamma\frac{\partial \ln \Sigma_0}{\partial \ln a}\right) - \frac{\partial}{\partial a}(aZ)\right] \\
U &= -X^2 + aZ\frac{\partial X}{\partial a} + Z^2\frac{\partial \ln \Sigma_0}{\partial \ln a} + \frac{A}{\Omega_B} \\
V &= \frac{B}{\Omega_B}\exp\left(-\int X d\xi\right)
\end{aligned}$$

Eigen Waves n nodes between inner and outer edge



Eigen waves with amplitude ← Force Term

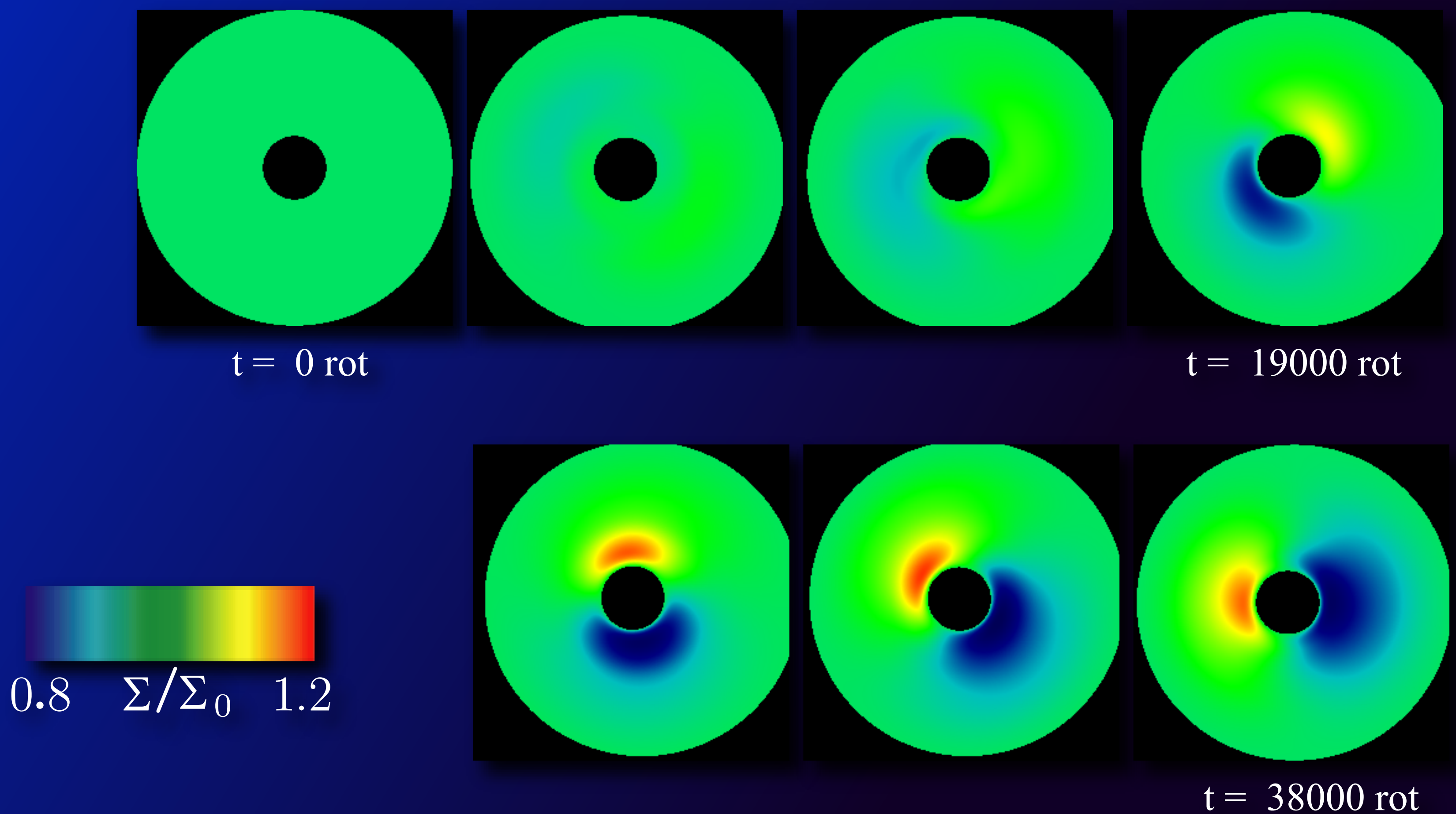


Density Evolution of flat Circumbinary Disk

Orbital Element → Density

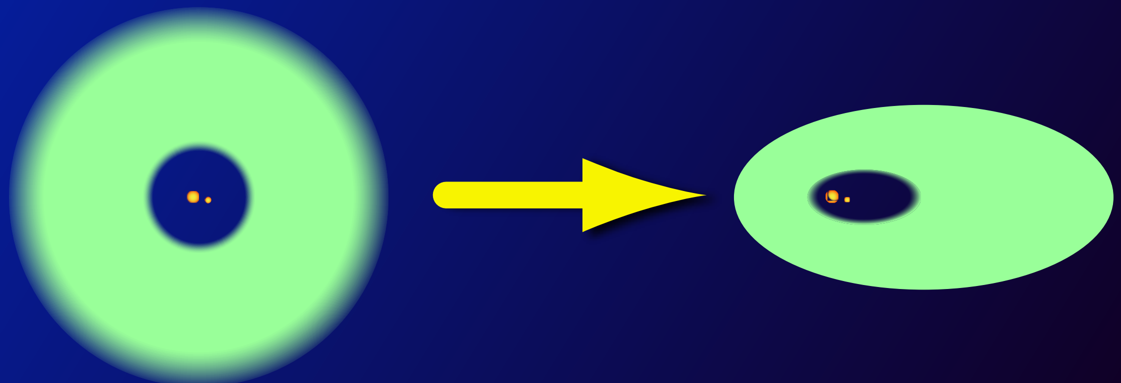
$M_1 : M_2 = 0.8 : 0.2 \quad e_B = 0.3 \quad \gamma_{\rm disk} = 0.0 \quad \gamma = 5/3$

$a_{\rm in} = 3.0 \quad a_{\rm out} = 15.0 \quad cs_{\rm in} = 0.07$



Secularly rotate m=1 density structure

Secular interaction with eccentric binary cause the circumbinary disk eccentric

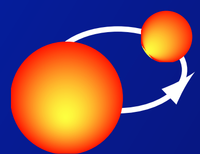


Does it changes the accretion picture?

Then, we calculate the gas accretion from the eccentric disk to the eccentric binary by SPH.

Initial Model

Binary Model



Binary Mass Ratio $M1 : M2 = 4 : 1$

Binary Eccentricity $e_B = 0.3$

Disk Model

$$\Sigma(a) = \frac{1}{4\sqrt{1-e_D^2}}\left(1+\tanh\frac{a-a_{in}}{\Delta_{in}}\right)\left(1+\tanh\frac{a_{out}-a}{\Delta_{out}}\right)$$

inner semimajor axis $a_{in} = 4$ $\Delta_{in} = 0.1\, a_{in}$

outer semimajor axis $a_{out} = 16$ $\Delta_{out} = 0.1\, a_{out}$

Disk eccentricity $e_D = 0.0 \sim 0.7$
spatially constant for one disk model

Polytrope $\gamma = 1.4$

Disk Gravity : Negligible

$c_S / v_K = 0.05$ at $a = 10$

Method

Numerical Method：2D-Godunov SPH
Particle Numbers : 65000 ~ 250000

Density

$$\Sigma_i = \sum_j m_j W(\boldsymbol{x}_j - \boldsymbol{x}_i, h_i) \qquad h_i = \eta \left(\frac{m_i}{\Sigma_i} \right)^{\frac{1}{2}}$$

We calculate the surface density by local summation of the particle mass.

We prefer the conservation property of the total mass for the long-term evolution.

Equation of Motion

$$\frac{d\boldsymbol{v}_i}{dt} = -\sum_j m_j \frac{1}{\pi h_{ij}^2} \frac{2P^*}{\Sigma_{ij}^2} \frac{2(\boldsymbol{x}_j - \boldsymbol{x}_i)}{h_{ij}^2} \exp\left[-\frac{(\boldsymbol{x}_j - \boldsymbol{x}_i)^2}{h_{ij}^2} \right]$$

The system is quite cold. (M>10, typically).
We include the Godunov type dissipative process in the calculation to capture shock.

Polytrope

$$P \propto \Sigma^\gamma$$

Polytropic relation is assumed for simplicity.

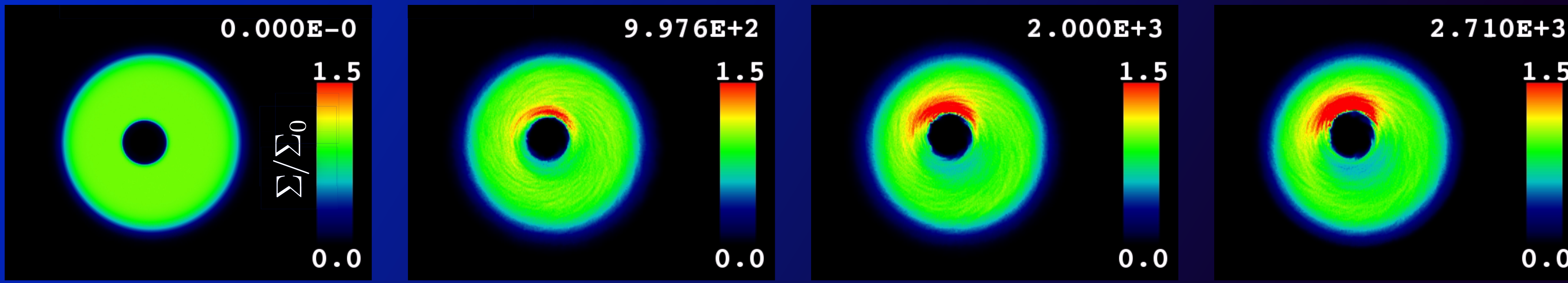
Particle Rezoning

$$\boldsymbol{x}_i \rightarrow \boldsymbol{x}_i - \sum_j \frac{h_1}{2\sqrt{\pi}} \frac{\boldsymbol{x}_j - \boldsymbol{x}_i}{|\boldsymbol{x}_j - \boldsymbol{x}_i|} \frac{m_j}{\Sigma_{ij}} W(\boldsymbol{x}_j - \boldsymbol{x}_i, h_2)$$

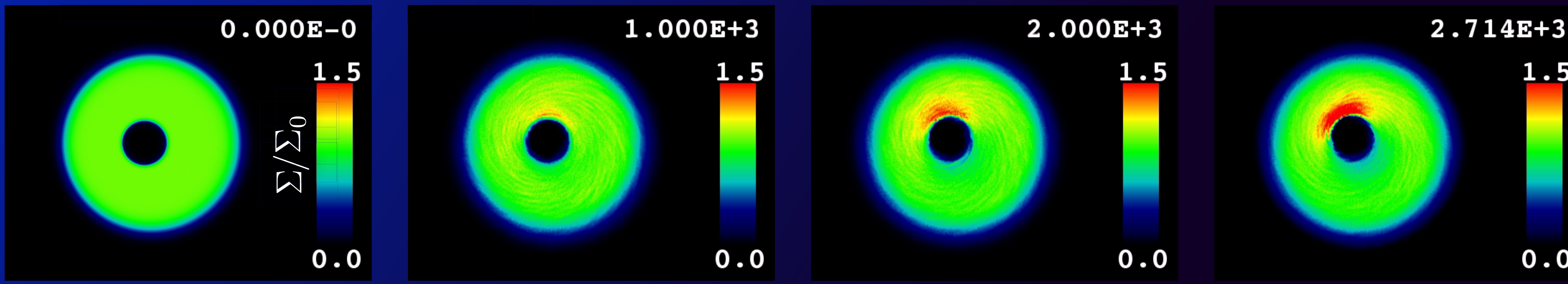
The system is cold shear and the density problem arises in such system.

To reduce the density error problem, we include the Particle Rezoning Method in the calculation.

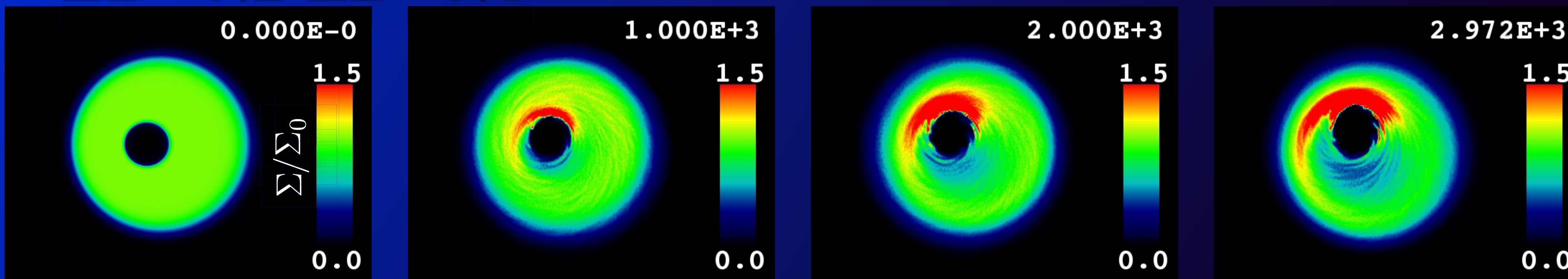
ED=0.1 EB=0.3



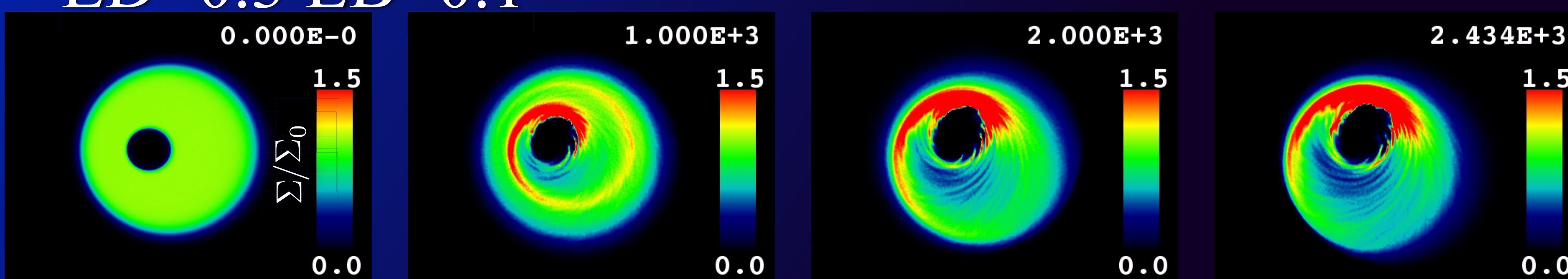
ED=0.1 EB=0.1

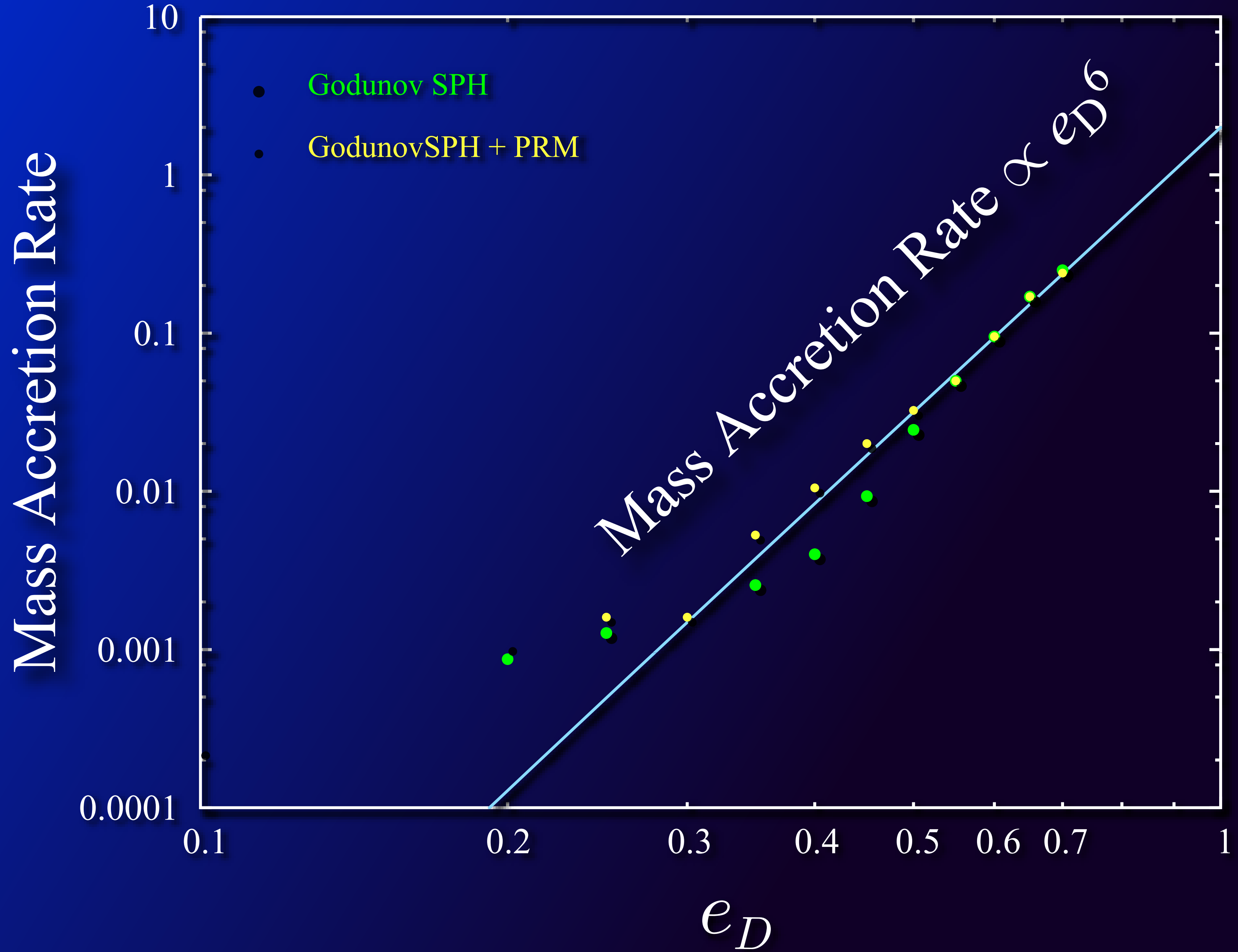


ED=0.2 EB=0.1



ED=0.3 EB=0.1




10
1
0.1
0.01
0.001
0.0001
0.1
0.2
0.3
0.4
0.5
0.6
0.7
1
Godunov SPH
GodunovSPH + PRM
Mass Accretion Rate ∝ $e_D{}^6$
Mass Accretion Rate
e_D

Conclusion

Secular interaction with eccentric binary sometimes causes the large eccentric disk.

When the disk becomes eccentric, m=1 density pattern can be formed.

Eccentric disk drastically changes the mass accretion rate.