# Information from dust continuum data

# Estimation of temperature, spectral index, and column density

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Sub-millimetre continuum data provide information on the column density and dust properties of interstellar clouds. We have compared methods that can be used to derive high-resolution column density maps from, e.g., Herschel measurements. We also have investigated the estimation of dust colour temperature and emissivity spectral index. Radiative transfer models are used to study the differences between the true and apparent cloud properties and to compare the performance of analysis methods.

Models show that, because of the nature of spatial temperature variations, externally heated clouds tend to show an artificial positive correlation between colour temperature and spectral index. For clouds with internal heating, the situation is reversed. Analysis of observations is affected by observational noise that also can produce a negative correlation. We find that, compared to direct least squares fitting of modified black body spectra, Bayesian methods and hierarchical statistical models are more accurate although not completely unbiased. Hierarchical models can mask possible local variation in the relation between temperature and spectral index.

Palmeirim et al. (2013) derived high-resolution column density maps using a combination of estimates obtained in different wavelength ranges. The method is quite reliable although somewhat sensitive to noise. For clouds with a simple density structure, radiative transfer modelling provides the most accurate estimates. As a simpler alternative, we propose modelling that consists of high-resolution column density and temperature maps that are matched to observations through convolution. The method is able to produce reliable column density estimates even at super-resolution. The method is computationally demanding but still feasible even in the analysis of large Herschel maps.

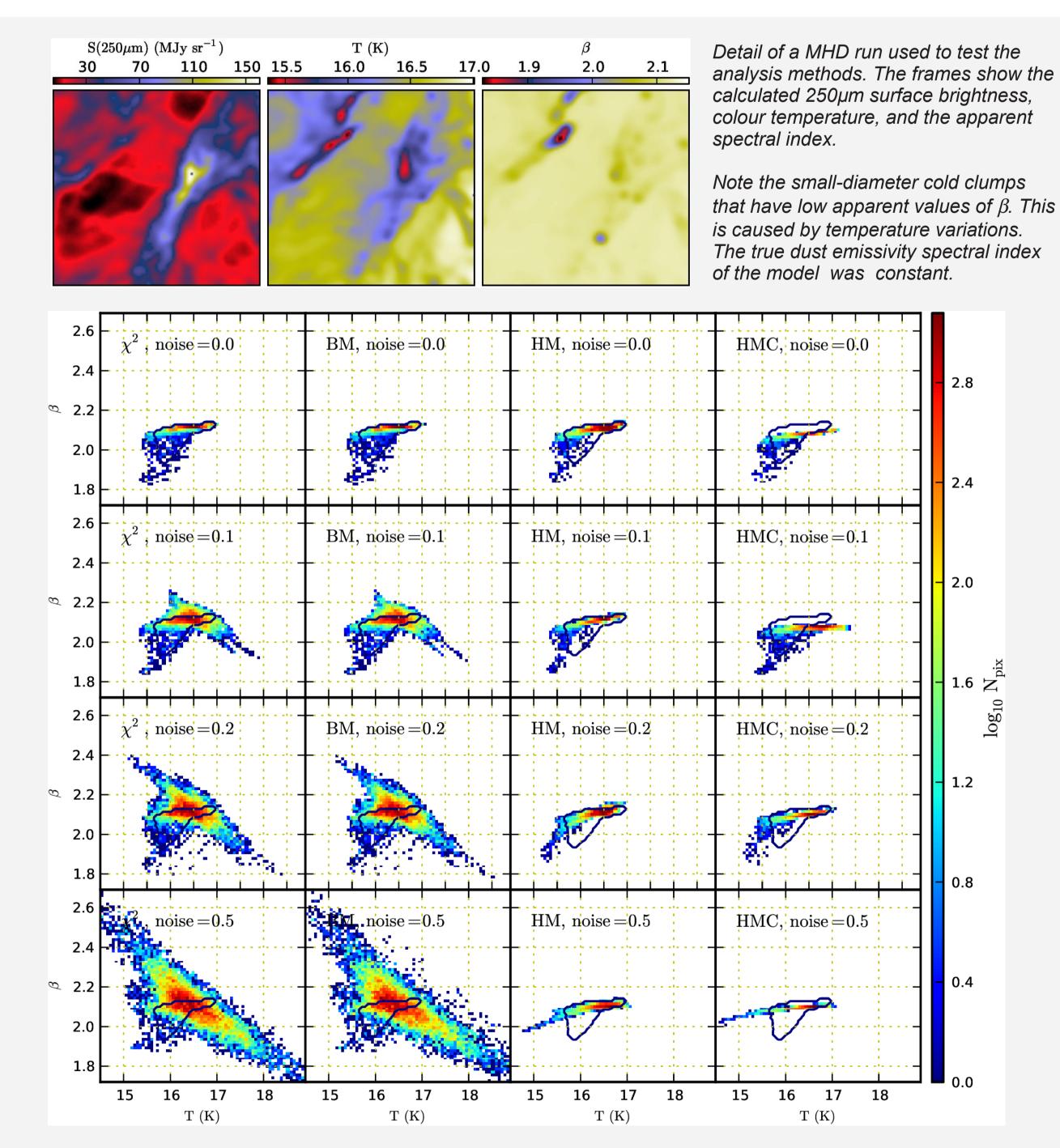
### The degeneracy between dust colour temperature and spectral index: Comparison of methods for estimating the $\beta(T)$ relation [2013a]

Submillimetre dust emission provides information on the physics of interstellar clouds and dust, but noise can produce a spurious anticorrelation between the colour temperature  $T_c$  and the emissivity spectral index  $\beta$ . These artefacts must be separated from the intrinsic  $\beta(T)$  relation. We compare methods that can be used to estimate  $(T, \beta)$  values and to analyse the  $\beta(T)$  relation.

We use submillimetre observations that are simulated either as simple, modified black body emission or using 3D radiative transfer modelling. We use several alternative methods to recover the  $(T, \beta)$  values of individual objects and the parameters of the overall  $\beta(T)$  relation. In addition to  $\chi^2$  fitting, we examine the results of the SIMEX method, Bayesian model (see Veneziani et al. 2010, 2012), hierarchical statistical models (see Kelly et al. 2012; this includes a model for the distribution of T and  $\beta$  values), and a method that explicitly assumes a functional form for  $\beta(T)$ . The methods were also applied to one field observed by Herschel.

All methods exhibit some bias, even in the idealised case of white noise. SIMEX did not perform reliably but Bayesian method shows significantly lower bias than direct  $\chi^2$  fits. The same is true for hierarchical models that also result in a smaller scatter in the temperature and spectral index estimates. However, significant bias was observed in cases with high noise levels. When the signal-to-noise ratios are different for different sources (or map pixels), all  $\beta$  and T estimates of the hierarchical model are biased towards the relation determined by the data with the highest signal-to-noise ratio. This can significantly alter the recovered  $\beta(T)$  function. With the method where we explicitly assume a functional form for the  $\beta(T)$  relation, the bias is similar to the Bayesian method but the method is slower. In the analysis of a Herschel field, all methods agree on similar  $T - \beta$  anticorrelation.

The Bayesian method and hierarchical models reduce the noise-induced parameter correlations. However, all methods can exhibit non-negligible bias. This is particularly true for hierarchical models and observations of varying signal-to-noise ratios. This must be taken into account when interpreting the results. Hierarchical models can be developed further to accommodate cases where data contain a small number of pixels whose behaviour deviates from most of the data.



The distributions of  $(T,\beta)$  values recovered with different methods. The data consists of synthetic observations of a turbulent MHD model cloud at 160 $\mu$ m, 250 $\mu$ m, 350 $\mu$ m, and 500 $\mu$ m. The rows correspond to gradually increased levels of observational noise. The columns correspond to methods: normal  $\chi^2$  fits, Bayesian method (BM), hierarchical statistical model assuming Gaussian statistics (HM), and similar hierarchical method that also fits for calibration errors (HMC). To guide the eye, a single contour following the solution for the noiseless case is plotted in each frame. The colour scale shows the logarithmic number of pixels per a  $(T,\beta)$  interval.

Note the difference in the noise of the recovered values and, for hierarchical models, the gradual disappearance of the

#### References:

Juvela M., Montillaud J., Ysard N., Lunttila T. (2013a) A&A (in press) Juvela M.; Malinen J.; Lunttila T. (2013b) A&A 553, A113 Juvela M., Montillaud J. (2013c) submitted Kelly B. et al. (2012) ApJ 752, 55

 $low-\beta$  pixels when the observational noise is increased.

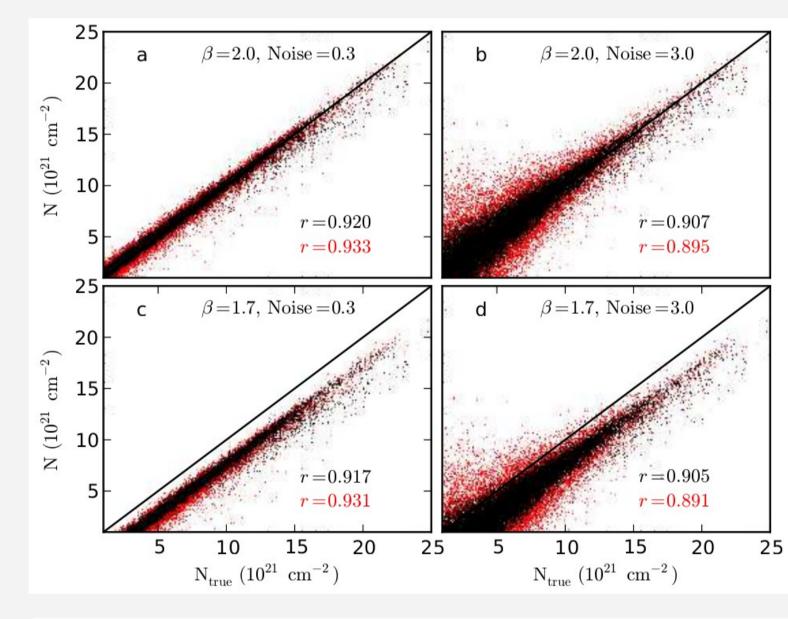
Palmeirim P. et al (2013) A&A 550, A38
Regarding *T*-β relation, see also: Juvela et al. (2012) A&A 541, A33; Juvela et al. (2012) A&A 539, A&A 71;
Ysard et al. (2012) A&A 542, A21; Malinen et al. (2011) A&A 530, 101

## Estimation of high-resolution dust column density maps Comparison of modified black-body fits and radiative transfer modelling [2013b]

Sub-millimetre dust emission is routinely used to derive the column density *N* of dense interstellar clouds. The observations consist of data at several wavelengths but also, with increasing wavelength, of poorer resolution. Procedures have been proposed for deriving higher resolution maps of N. We compared two alternatives. Method A uses low-resolution temperature estimates combined with higher resolution intensity data while Method B combines column density estimates obtained using different wavelength ranges.

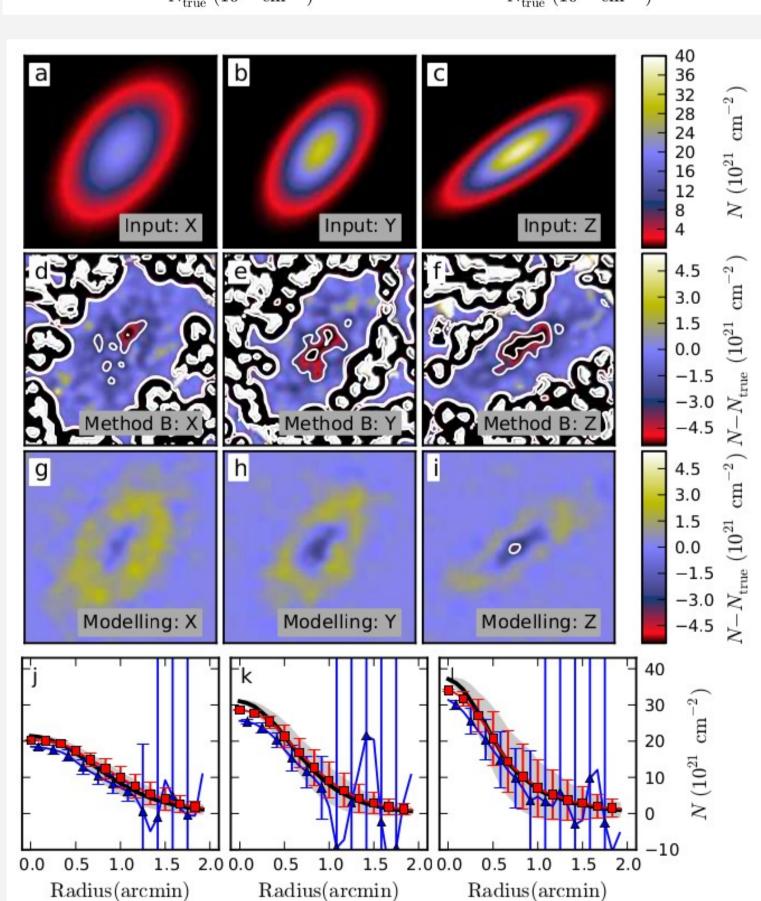
We used MHD simulations and radiative transfer calculations to simulate sub-millimetre surface brightness observations at the wavelengths of the Herschel Space Observatory. The synthetic observations were analysed with the two methods and the results were compared to the true column densities and to the results from with simple 3D radiative transfer modelling.

Both methods give relatively reliable column density estimates at the resolution of 250 µm data while also making use of the longer wavelengths. For high signal-to-noise ratios, the results of Method B are better correlated with the true column density, while Method A is less sensitive to noise. When the cloud has internal heating sources, Method B gives results that are consistent with those that would be obtained if high-resolution data were available at all wavelengths. Because of line-of-sight temperature variations, these underestimate the true column density. Because of a favourable cancellation of errors, Method A can sometimes give more correct values. Radiative transfer modelling, even with very simple 3D cloud models, usually provides more accurate results. However, the complexity of the models that are required for improved results increases rapidly with the complexity and opacity of the clouds.



Column densities estimated with Method A (black points) and Method B (red points) versus the true column density of a model cloud. The frames correspond to 0.3 or 3.0 times the assumed default noise. The numbers indicate the correlation coefficients r for data with  $N_{true} > 5 \times 10^{21}$  cm<sup>-2</sup> . For illustration of the  $\beta$  dependence, we have included in the lower frames the corresponding results obtained assuming a spectral index value of  $\beta$  = 1.7 instead of the correct  $\beta$  = 2.0.

Note the higher noise level of the Method B estimates.



Results for a simple ellipsoid cloud (see the paper for examples of more complex turbulent, self-gravitating clouds). The frames a-c show the true column densities towards three directions. The errors in the column density estimates of Method B and of the 3D modelling are shown in frames d– f and g–i, respectively, with contours drawn at  $-3 \times 10^{21}$  cm<sup>-2</sup> and  $-6 \times 10^{21}$  cm<sup>-2</sup>. The bottom frames show the radial column density profiles. The true column density is shown with a black solid line, the grey area indicating the 1  $\sigma$  variation in the averaged rings. The triangles correspond to Method B and the square symbols to the 3D modelling. The error bars indicate the corresponding 1 σ variation in the averaged rings.

Note that unlike Method B, the physical modelling (based on radiative transfer modelling with column densities optimised pixel-by-pixel) results in very low noise at low column densities.

#### High-resolution dust column density maps: Empirical model fits [2013c]

Continuing on the previous examination of column densities, we propose simple model fitting (Method E) as a flexible way to estimate high resolution column density maps. We have evaluated the accuracy of this procedure and we discuss its suitability for the making of column density maps at a resolution better than the worst resolution of the observed wavelengths.

The method consists of model maps of column density (or intensity at a reference wavelength) and colour temperature. The model is fitted using Markov Chain Monte Carlo (MCMC) methods, comparing model predictions with observations at their native resolution. We analyse simulated surface brightness maps and compare its accuracy with method B (see above) and the results that would be obtained using high-resolution observations without noise.

The new method is able to produce reliable column density estimates at a resolution significantly better than the lowest resolution of the input maps. Compared to method B, it is resilient against the effects of noise. The method is computationally more demanding but still feasible even in the analysis of large Herschel maps. It is a viable alternative for the calculation of high resolution column density maps, with potential even for considerable super-resolution. Both methods E and B include potential for further improvements, e.g., in the form of better a priori constraints.